

Answers to Math NYB-Final Exam (December 2011)

1.  $f'(x) = \frac{-1}{1 + (x + 1)^2}$

2. (a)  $\frac{2}{5}(x - 4)^{5/2} + \frac{16}{3}(x - 4)^{3/2} + 32(x - 4)^{1/2} + C$

(b)  $\frac{(\arcsin(x^2))^2}{4} + C$

(c)  $\frac{20}{21}$

(d)  $\frac{1}{2} \left( \theta + \frac{\sin(2\theta)}{2} \right) - \cos \theta + \frac{\cos^3 \theta}{3} + C$

(e)  $\sqrt{9x^2 - 4} - 2\operatorname{arcsec} \left( \frac{3x}{2} \right) + C$

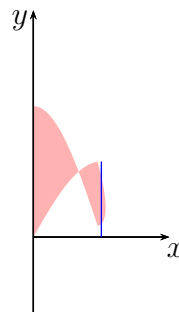
(f)  $\ln|x - 2| + \frac{5}{2} \ln(x^2 + 9) + \frac{5}{3} \arctan \left( \frac{x}{3} \right) + C$

(g)  $\frac{(16x^2 + 1) \arctan(4x)}{2} - 2x + C$

3. (a) 0      (b)  $\frac{1}{2}$       (c)  $e^2$

4. (a)  $\frac{\pi}{18}$       (b)  $\infty$  (diverges)

5.  $A = \int_0^{\pi/3} (\sqrt{3} \cos x - \sin x) dx + \int_{\pi/3}^{\pi/2} (\sin x - \sqrt{3} \cos x) dx = 3 - \sqrt{3}$



6.  $y = \frac{\sqrt{3 + \sqrt{x^2 + 1}}}{2}$

7. (a)  $V = 2\pi(6 - \ln 3)$ ,      (b)  $V = \pi \int_0^2 \left[ (3 + x^2)^2 - \left( 3 - \frac{1}{x+1} \right)^2 \right] dx$

8. 0

9. (a)  $a_1 = 1, a_2 = 2, a_3 = \frac{5}{2}, a_4 = \frac{13}{5}$

(b)  $a_n < 3$

(c) Monotonic Sequence Theorem

(d)  $\lim_{n \rightarrow \infty} a_n = \frac{3 + \sqrt{5}}{2}$

10. (a) Geometric series with  $r = -\frac{2}{3}$ ; it converges to  $S = -\frac{2}{45}$   
(b) Telescoping sum; it converges to  $\frac{3}{2}$
11. (a) diverges by Test for Divergence ( $N^{\text{th}}$  Term Test)  
(b) diverges by Integral Test  
(c) converges by Limit Comparison Test  
(d) converges by Ratio Test ( $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \frac{1}{e}$ )
12. (a) AC by Root Test  
(b) AC by Direct Comparison Test  
(c) CC
13.  $R = \frac{1}{e}$  and IoC =  $\left[4 - \frac{1}{e}, 4 + \frac{1}{e}\right]$
14.  $T_3(x) = 1 - 3x + 6x^2 - 10x^3$  and  $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (n+1)(n+2)}{2} x^n$