(Marks)

(3) 1. A scrabble player has seven tiles with a different letter from the English alphabet on each of them. How many different four-letter words (i.e. ordered strings of letters which may or may not have any meaning) can be made using these seven tiles?

(3) 2. Socks are sold in three colours: white, black and grey with pink polka-dots. If Tyler wants to purchase seven pairs of socks for the new year 2012, then how many different combinations of socks can he purchase?

(4) 3. Let A be a set with ten elements.

(a) How many subsets does A have?

(b) How many subsets of A have exactly eight elements?

(c) Determine how many subsets of A have fewer than eight elements. (Hint: Your answers from a) and b) should be helpful.)

(4) 4. Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2, 4\}$, $B = \{1, 4, 5, 6\}$, $C = \{x \in \mathbb{N} : 3x \leq 13\}$. List the elements of each set indicated.

(a) C

(b) $A \cap B$

(c) $(A \cup B) \cap (A \cup \phi)$

(d) $\overline{A} \cup (\overline{U \cap B})$

(2) 5. Write the set of all positive even integers in set-builder notation.

(4) 6. Given the statement P: "If I miss the bus then I am late for school."

(a) Write the converse of P.

(b) Write the inverse of P.

(c) Write the contrapositive of P.

(d) Are any of the three statements logically equivalent to the original statement P? If so, which one(s)?

(3) 7. Provide a truth table for the following statement. Determine whether the statement is a tautology, a contradiction or neither.

$$[p \to (q \veebar r)] \wedge (\sim p)$$

(3) 8. Use truth tables to determine whether or not the following two statements are equivalent.

a) $\sim p \to \sim q$ b) $p \vee \sim q$

(3) 9. Use Venn diagrams to prove this distributive law for sets: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. Be sure to clearly indicate which hatched area corresponds to each set.

(3) 10. Use Venn diagrams to determine whether or not the argument is valid.

H: Some apples are red.

H: No oranges are red.

C: No apples are oranges.

(4) 11. Use truth tables to determine whether or not the following argument is valid:

H: If general relativity is correct then neutrinos cannot travel faster than light.

H: Neutrinos can travel faster than light.

C: General relativity is not correct.

(3) 12. Given the Boolean expression: $(A + \overline{B})(B + \overline{C})(C + \overline{A})$

(a) Draw a network corresponding to the expression.

(b) Evaluate the expression when $A = 1$, $B = 1$ and $C = 0$.

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(5) 13. Simplify the following networks. Justify by citing any network properties used in simplification.

(a) $A(B+\overline{B})+\overline{A}+B$

(b) $(\overline{A+B})\overline{A}+A\overline{B}+C\overline{C}$

(2) 14. Provide a Boolean table for $(\overline{AB})(A+B)$.

(2) 15. Draw, then find a Boolean expression for, a network involving switches A, B, C and their complements such that current will pass through the network whenever exactly two of the switches A, B, C are closed. No current should pass if all three switches A, B, C are closed or if fewer than two of the three are closed.

(1) 16. State one of DeMorgan's laws for networks.

(1) 17. State the associative property for unions of sets.

(2) 18. Is matrix multiplication commutative? In other words, is it true that $AB = BA$ for any two matrices A, B? Justify your answer.

(4) 19. Without solving classify each system below as dependent or independent and as consistent with one solution only, consistent with infinitely many solutions or inconsistent. Justify your answers.

a) $\begin{aligned} 3x - y &= -2 \\ 6x - 2y &= -4 \end{aligned}$ b) $\begin{aligned} 2x - 5y &= 7 \\ -2x + 5y &= 7 \end{aligned}$

(5) 20. Given the system $\begin{cases} 3x + 5y = -1 \\ -x + 3y = 5 \end{cases}$

a) Graph both lines then estimate the solution to the system.
b) Solve the system algebraically by substitution or elimination.

(12) 21. Let $A = \begin{bmatrix} -1 & 3 \\ 5 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -8 \\ -1 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 1 & -3 & 2 \\ 7 & 1 & 0 \\ -2 & 4 & 5 \end{bmatrix}$

$D = \begin{bmatrix} 1 & -1 & 3 \\ -6 & 2 & 1 \end{bmatrix}$ $E = \begin{bmatrix} 1 & -3 \\ 2 & -1 \\ -2 & 0 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

find each of the following, if possible. If an operation is not possible, say why.

a) $C^T - 3I$ b) A^{-1} c) B^{-1}
d) A^2 e) $3D + E$ f) $B^T E$
g) EA

(6) 22. If it exists, find the inverse of the matrix $A = \begin{bmatrix} 4 & -9 & 14 \\ 1 & -2 & 3 \\ -2 & 6 & -9 \end{bmatrix}$ and verify that your answer is correct. If no inverse exists, explain why this is the case.

(5) 23. Given the linear system: $\begin{cases} -4x - 3y + 2z = 2 \\ 4x + 3y - z = -1 \\ -3x - 2y + z = 3 \end{cases}$

a) Write the system in matrix form $AX = B$.

b) If $A^{-1} = \begin{bmatrix} 1 & -1 & -3 \\ -1 & 2 & 4 \\ 1 & 1 & 0 \end{bmatrix}$, solve the system using A^{-1}.

(4) 24. Each matrix below is the *augmented* matrix for a corresponding system of equations. Solve the system corresponding to each matrix.

a) $A = \left[\begin{array}{ccc|c} 1 & -3 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$ b) $A = \left[\begin{array}{ccc|c} 1 & 0 & 5 & -5 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right]$

(Marks)

(5) 25. Solve the following system of equations using matrices and row operations. Verify that your answer is correct.

$$\begin{aligned} 3x - 5y + 14z &= -2 \\ 2x - 4y + 7z &= 5 \\ x - 2y + 3z &= 4 \end{aligned}$$

(4) 26. Solve the following system of equations using matrices and row operations. Verify that your answer is correct.

$$\begin{aligned} x + 2y - 3z &= -1 \\ 3x + 8y - 13z &= -9 \\ 5x + 12y - 19z &= -11 \end{aligned}$$

(3) 27. Use mathematical induction to prove that the following statement holds for all positive integers n.

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$