

(Marks)

1. Evaluate each of the following integrals without the use of integration tables.

(3) (a)
$$\int \frac{\sqrt[4]{x} + x^3 e^x - 2x^3 \csc^2 x + 3x^3 \cdot 5^x}{3x^3} dx$$

(3) (b)
$$\int_0^1 \frac{x^5 + 4x^4 - x^2 + 20x}{x^2 + 4x} dx$$

(3) (c)
$$\int \frac{x^2 - 2x - 6}{x^3 + 3x^2} dx$$

(3) (d)
$$\int_2^e \frac{2}{x (\ln x)^6} dx$$

(3) (e)
$$\int (x^2 - 5) e^x dx$$

(3) (f)
$$\int x^3 \ln(2x) dx$$

(4) 2. Given the curves $f(x) = 5x - x^2$ and $g(x) = 3x^2 + x$, determine,(a) the point(s) of intersection of $f(x)$ and $g(x)$.(b) the area bounded by $f(x)$ and $g(x)$.(4) 3. Use Trapezoidal rule with $n = 5$ to approximate $\int_1^6 \frac{e^{x-1}}{\sqrt{x+1}} dx$.

Round your answer to four decimal places.

(6) 4. Given the demand function $p = -x^2 + 24$ and the supply function $p = x^2 + 2x$,

(a) find the equilibrium point.

(b) sketch and identify the regions representing the consumer and producer surplus.

(c) evaluate the producer surplus.

5. Use integration tables to integrate the following.

In each case, state the formula number and justify its use.

(4) (a)
$$\int \frac{\sqrt{11 + 10x - x^2}}{x - 5} dx$$

(4) (b)
$$\int \frac{(x+2)^2}{\sqrt{x+7}} dx$$

(4) 6. Determine if the function $y = e^x \sin x$ is a solution for the differential equation $y'' - 2y' + 3y = e^x \sin x$

(4) 7. The population of a certain bacteria culture increases at a rate proportional to its population and inversely proportional to the square root time. The initial population of 500 increases by 30% in 2 hours. What will be the population of the bacteria culture in 21 hours?

8. Solve the following differential equations for y :

(4) (a)
$$y' = \frac{xy}{\sqrt{4+x^2}}$$
 with initial condition $y(0) = 1$

(4) (b)
$$\csc(x) y' = \frac{x}{e^y}$$
 with initial condition $y(0) = 1$

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9. Evaluate each improper integral and state whether it converges or diverges. In the case where the integral diverges indicate whether it diverges to $-\infty$ or $+\infty$.

(4) (a) $\int_1^3 \frac{1}{\sqrt{3-x}} dx$

(4) (b) $\int_e^\infty \frac{\ln x}{x} dx$

10. Use l'Hôpital's rule to evaluate the following limits

(4) (a) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan(x - \frac{\pi}{2})}{\cos(x)}$

(4) (b) $\lim_{x \rightarrow 0} \frac{\ln(\sin x + 1)}{1 - e^{x/3}}$

(3) 11. Consider the sequence $\left\{ \frac{-3}{2}, \frac{5}{5}, \frac{-7}{10}, \frac{9}{17}, \dots \right\}$

(a) What is the value of the 5th term?

(b) Find an expression for the n^{th} term of the sequence.

12. Determine the convergence or divergence of each sequence $\{a_n\}$.

If the sequence converges, find the limit. In the case where the integral diverges indicate whether it diverges to $-\infty$ or $+\infty$.

(4) (a) $a_n = \frac{n!}{2^n(n-1)!}$

(4) (b) $a_n = \frac{-n^3 + 5}{n\sqrt{n^3 + 7n}}$

- (3) 13. An amount of \$3500 is deposited in a GIC that pays 4.25% interest compounded quarterly. Find the balance at the end of 10 years.

14. Determine with justification if each of the following series is convergent or divergent.

If the series is convergent, find its sum.

(3) (a) $\sum_{n=1}^{\infty} \frac{2^{n+1}}{7^n}$

(3) (b) $\sum_{n=1}^{\infty} \frac{(2-n)^2}{n^2 + 3n - 1}$

- (4) 15. A deposit of \$200.00 is made at the beginning of each month into a saving account that pays 4% interest a year, compound monthly. Find the balance after 5 years.

- (4) 16. Given a repeated decimal $1.\overline{407}$, express it using a geometric series, find the sum of the geometric series and write the decimal as the ratio of two integers.

(Marks)

ANSWERS

(1 a) $\frac{-4}{21x^{7/4}} + \frac{e^x}{3} + \frac{2\cot x}{3} + \frac{5^x}{\ln(5)} + C$; (1 b) $-\frac{3}{4} + 24 \ln\left(\frac{5}{4}\right) \approx 4.6054$

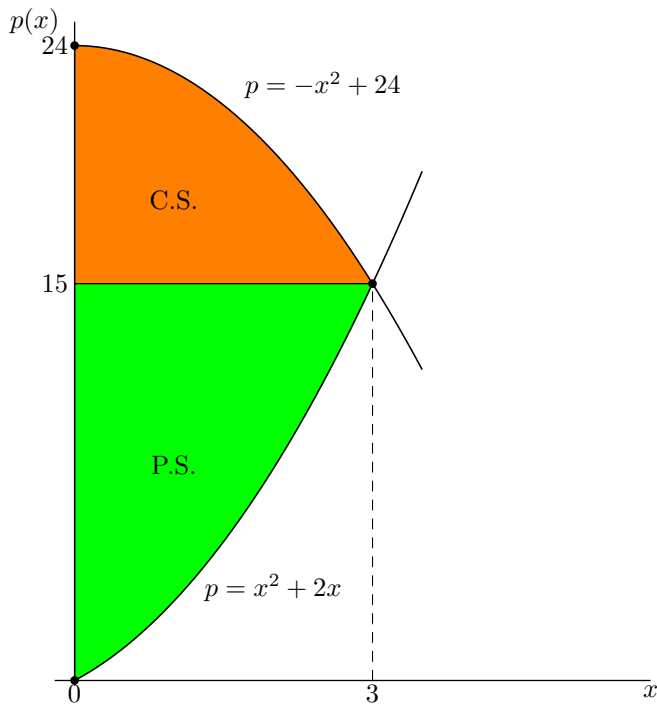
(1 c) $\frac{2}{x} + \ln|x+3| + C$; (1 d) $-\frac{2}{5} + \frac{2}{5[\ln(2)]^5} \approx 2.1$; (1 e) $x^2e^x - 2xe^x - 3e^x + C$

(1 f) $\frac{x^4}{4} \ln(2x) - \frac{x^4}{16} + C$

(2 a) (0, 0) and (1, 4); (2 b) $\frac{2}{3} \approx 0.67$ square units; (3) 64.9371

(4 a) point of equilibrium: (3, 15)

(4 b)



(4 c) P.S. = \$ 27.00

(5 a) complete the square then F21: $\sqrt{11 + 10x - x^2} - 6 \ln \left| \frac{6 + \sqrt{11 + 10x - x^2}}{x - 5} \right| + C$

(5 b) substitution then F16: $\frac{2}{15} \left(3(x+2)^2 - 20(x+2) + 200 \right) \sqrt{x+7} + C = \frac{2}{15} (3x^2 - 8x + 172) \sqrt{x+7}$

(6) it is a solution; (7) the population is 1170

(8 a) $y = e^{(4+x^2)^{1/2}-2} = \frac{e^{(4+x^2)^{1/2}}}{e^2}$; (8 b) $y = \ln|-x \cos x + \sin x + e|$

(9 a) $2\sqrt{2} \approx 2.83$; (9 b) $+\infty$; (10 a) -1 ; (10 b) -3 ; (11 a) $a_5 = -\frac{11}{26}$; (11 b) $a_n = (-1)^n \frac{2n+1}{n^2+1}$

(12 a) converges to 0; (12 b) diverges; (13) \$5341.58; (14 a) $\frac{4}{5} = 0.8$; (14 b) divergent

(15) \$13302.60; (16) $\frac{38}{27}$