

(Marks)

1. Evaluate the following limits.

(3) (a)  $\lim_{x \rightarrow \pi} \frac{\sin^2 x}{1 + \cos(3x)}$

(3) (b)  $\lim_{x \rightarrow \infty} x^2 \ln \left( 1 + \frac{4}{x^2} \right)$

2. Evaluate the following integrals.

(5) (a)  $\int_1^5 \frac{x+2}{\sqrt{2x-1}} dx$

(5) (b)  $\int \frac{1}{x^3 \sqrt{x^2-4}} dx$

(5) (c)  $\int \frac{\tan^{-1} x}{x^2} dx$

(5) (d)  $\int \frac{\sec^4 \sqrt{x} \tan^2 \sqrt{x}}{\sqrt{x}} dx$

(5) (e)  $\int_0^{\frac{1}{2}} \frac{x + \arccos x}{\sqrt{1-x^2}} dx$

(5) (f)  $\int \frac{e^x}{\sqrt{3-2e^x-e^{2x}}} dx$

(5) (g)  $\int \frac{3x^2-2}{x^2-2x-8} dx$

(3) 3. Find the area enclosed by the curves  $y = \sqrt{x-1}$ ,  $y = x-1$ , and  $x = 5$ .4. Let  $\mathcal{R}$  be the region bounded by  $y = 2e^x$  and  $y = 2 \ln x$ , between  $x = 1$  and  $x = e$ .(4) (a) Find the volume of the solid obtained by rotating the region  $\mathcal{R}$  about the  $y$ -axis.(3) (b) Set up, but do not attempt to calculate, the integral for the volume of the solid obtained by rotating the region  $\mathcal{R}$  about the horizontal line  $y = -1$ .

5. Evaluate each of the following improper integrals.

(5) (a)  $\int_{\frac{1}{2}}^{\infty} \frac{\tan^{-1}(2x)}{4x^2+1} dx$

(5) (b)  $\int_0^6 \frac{1}{(x-2)^3} dx$

(4) 6. Solve the differential equation  $y' = 1 + x^2 + y^2 + x^2 y^2$  with  $y(0) = 1$ . Express  $y$  as a function of  $x$ .(3) 7. Find the sum of  $\sum_{n=1}^{\infty} \frac{2}{(2n-1)(2n+1)}$ 8. Find the limit  $\lim_{n \rightarrow \infty} a_n$  for each sequence.

(2) (a)  $a_n = \frac{\sin(n!)}{n+1}$

(2) (b)  $a_n = \frac{(-1)^n 3^{n+2}}{2^{2n+1}}$

(Marks)

(2) 9. Provide an example of a geometric series whose sum is  $\frac{\pi}{12}$ .

10. Determine whether the following series are convergent or divergent.

(3) (a)  $\sum_{n=1}^{\infty} \left(1 - \frac{2}{3n^2}\right)^n$

(3) (b)  $\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}$

(3) (c)  $\sum_{n=1}^{\infty} \frac{\sqrt{\arctan(n)}}{1+n^2}$

11. Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent.

(3) (a)  $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt[3]{n^3+1}}{1+n+n^3}$

(3) (b)  $\sum_{n=1}^{\infty} \frac{(-2)^n}{(\ln n)^n}$

(3) (c)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+3}}$

(4) 12. Find the interval of convergence of the power series  $\sum_{n=0}^{\infty} \frac{(x+1)^n}{5^n \sqrt[3]{n+1}}$ (4) 13. Find the Maclaurin series for  $f(x) = \ln(x+2)$ , and its interval of convergence.**Answers**

1.(a)  $\frac{2}{9}$  (b) 4 2.(a)  $\frac{28}{3}$  (b)  $\frac{1}{16} \left( \sec^{-1} \left( \frac{x}{2} \right) + \frac{2\sqrt{x^2-4}}{x^2} \right) + C$

(c)  $-\frac{\tan^{-1} x}{x} + \ln x - \frac{1}{2} \ln(x^2+1) + C$  (d)  $2 \left( \frac{\tan^5 \sqrt{x}}{5} + \frac{\tan^3 \sqrt{x}}{3} \right) + C$  (e)  $1 - \frac{\sqrt{3}}{2} + \frac{5\pi^2}{72}$

(f)  $\arcsin \left( \frac{e^x+1}{2} \right) + C$  (g)  $3x + \frac{23}{3} \ln|x-4| - \frac{5}{3} \ln|x+2| + C$  3. 3 4.(a)  $4\pi[e^e(e-1) - \frac{e^2}{4} - \frac{1}{4}]$

(b)  $\int_1^e [\pi(1+2e^x)^2 - \pi(1+2\ln(x))^2] dx$  5.(a)  $\frac{3\pi^2}{64}$  (b)  $\infty$  6.  $y = \tan(x + \frac{x^3}{3} + \frac{\pi}{4})$  7. 1

8.(a) 0 (b) 0 9.  $\sum_{n=1}^{\infty} \frac{\pi}{24} \left( \frac{1}{2} \right)^{n-1}$  10.(a) Div. (b) Div. (c) Con. 11.(a) AC (b) AC

(c) CC 12.  $[-6, 4)$  13.  $\ln 2 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n 2^n}$  with IoC  $(-2, 2]$