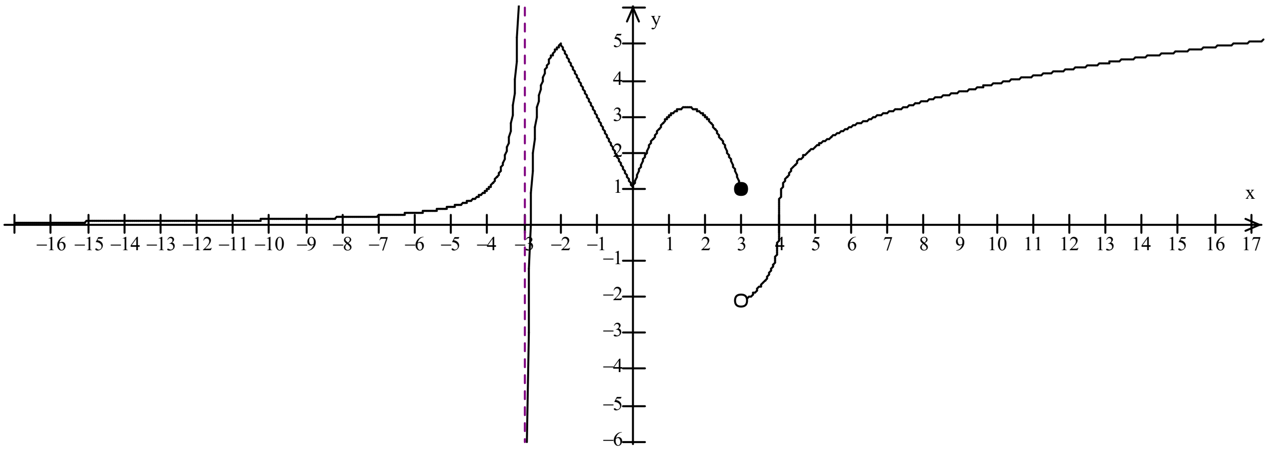
201-NYA-05 May 2011 Final Exam

1. Use the graph of the function *f(x)* below to determine the following.



   a. List the x value(s) at which the function f(x) is non differentiable
   b. List the x value(s) at which the function f(x) is continuous but non differentiable

2. Evaluate the following. Use $+\infty, -\infty$ or "does not exist" where appropriate.

   a. $\lim\limits_{x\to 5} \dfrac{x^2-7x+10}{3x-15}$

   b. $\lim\limits_{x\to -\infty} \dfrac{6x}{\sqrt{9x^2-5x}}$

   c. $\lim\limits_{x\to 10} \dfrac{\frac{1}{5}-\frac{2}{x}}{x-10}$

   d. $\lim\limits_{x\to 0} \dfrac{\sqrt{9+3x}-\sqrt{9}}{2x}$

   e. $\lim\limits_{x\to 0^-} \dfrac{\sin x - \frac{1}{\csc x} + e^x}{2\sec^2 x + \frac{|x|}{x}}$

3. Given $f(x) = \begin{cases} 3\cos(x\pi)-10 & \text{if } x<1 \\ kx-6 & \text{if } x\geq 1 \end{cases}$ find all value(s) of *k* which make *f(x)* continuous.

4. a) State the Quotient Rule

   b) Prove the quotient rule for $y = \dfrac{f(x)}{g(x)}$ (you may use logarithmic differentiation)

5. Does the function $g(x) = x^3 + 3x^2 + 6x - 3$, have a zero in the open interval (0, 1)? Explain your answer.

6. Given the function $f(x) = \sqrt{3x-1}$, find $f'(x)$ using the LIMIT DEFINITION of the derivative.

7. Find $\frac{dy}{dx}$ for each of the following: (Do not simplify your answers)

a. $y=\sqrt[5]{x^3}+5^x+\frac{1}{3x}+x^3+\csc x-\sin\left(\frac{\pi}{2}\right)$

b. $y=\sin^2\left(x^4+3\right)-\sec(\cot x)$

c. $y=\left(x^2-9\right)^{\sin x}$

d. $y=e^x\sec(3x^2-9)$

e. $y=\ln\sqrt[3]{\frac{x}{(x+4)^5\,(x^2-9)}}$

8. Find the equation of the normal line to the curve $y=x^3-4x-3$ at the point $x=-2$

9. Given the $x^2+xy+y^2=3$;

a. Find $\frac{dy}{dx}$

b. Determine all points $(x\,,\,y)$ on the curve where the tangent line is parallel to the line $y=x+5$

10. Determine all value(s) of x where the curve $y=x^3\ln\left(x^2\right)$ has horizontal tangents.

11. A paper cup has the shape of a cone with height 20 *cm* and radius 5 *cm* (at the top). If water is being poured into the cup at a rate of 3 cm^3/s , how fast is the water level rising when the water is 2 *cm* deep? *(You may find the following formulas useful. The volume of a cone is* $\frac{1}{3}\pi r^2 h$ *.)*

12. Given $f(x)=\frac{x}{4-x^2}$, $f'(x)=\frac{4+x^2}{\left(4-x^2\right)^2}$, $f''(x)=\frac{2x\left(12+x^2\right)}{\left(4-x^2\right)^3}$,find:

a. The domain of *f(x)*.
b. The x and y intercepts.
c. The vertical asymptote(s).
d. $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
e. The interval of increase and decrease.
f. All the local (relative) extrema (if any).
g. The interval of upward and downward concavity.
h. All the inflection point(s) (if any).
i. Sketch the graph of *f(x)*. Label all intercepts, asymptotes, extrema and points of inflection.

13. For the function defined by $f(x)=x^{2/3}-x+1$, find:

a. $f(-1)$ and $f\left(\frac{125}{8}\right)$ (give exact answers using the fact that $2^3=8$ *and* $5^3=125$)

b. The absolute maximum and absolute minimum of f(x) on the interval $\left[-1\,,\,\frac{125}{8}\right]$

14. A closed cylindrical can of volume 1 m^3 is to be designed to minimize cost. The cost of the top and bottom of the can is 80 cents per square meter, and the cost of the side is 50 cents per square meter.

Find the dimensions of the can (the radius and height) which minimize cost.

(You may find the following formulas useful. The area of a circle with radius r is πr^2. The volume of a cylinder with radius r and height h is $\pi r^2 h$. The surface area of a cylinder, excluding the top and bottom is $2\pi r h$.)

15. Find the position function $s(t)$ of a moving particle which has an acceleration function $a(t) = 3t + 3\cos t$, an initial velocity of $v(0) = 4\ m/s$ and an initial position of $s(0) = 3\ m$.
16. Evaluate the following integrals.

    a. $\int_1^e \frac{x-3}{x}\,dx$

    b. $\int \sqrt[3]{x^2} + 2\sin x - e^2\,dx$

    c. $\int \sin^2 x + \cos^2 x - \frac{1}{\sec x}\,dx$
17. Find the area bounded by $y = \frac{1}{2}e^x$, $x = -1$, $x = 3$, and the x axis
18. Let S be the region bounded by $f(x) = x^2 + 1$ and the x axis between $x = 0$ and $x = 2$. Approximate the value of the area S by finding the Riemann sum with 4 equal subintervals, taking the sample points to be the Right Endpoints.
19. Let $F(x) = \int_{-3}^{x^3} e^{t^3}\,dt$, evaluate $F'(x)$ using the fundamental theorem of calculus.

Answers:

1) a) -3,-2,0,3 and 4 b) -2,0, and 4
2) a) 1 b) -2 c) 1/50 d) ¼ e) 1
3) k = -7
4) a) $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ b) if $y = \frac{f}{g}$, use logarithmic differentiation $\ln y = \ln\left(\frac{f}{g}\right)$ to prove the quotient rule
5) g(x) is continuous on the interval [0,1], and g(0)=-3 <0 while g(1)=7 >0.
   Since -3<0<7 , there is a number c in [0,1] such that g(c)=0 by the intermediate value theorem. Thus there is a root of g(x) in the interval (0, 1)
6) $f'(x) = \frac{3}{2\sqrt{3x-1}}$
7) a) $y' = \frac{3}{5x^{2/5}} + 5^x \ln 5 - \frac{1}{3x^2} + 3x^2 - \csc(x)\cot(x)$

   b) $y' = 8x^3 \sin(x^4+3)\cos(x^4+3) + \sec(\cot(x))\tan(\cot(x))\csc^2(x)$

   c) $y' = \left(x^2-9\right)^{\sin(x)}\left[\cos(x)\ln(x^2-9) + \frac{2x\sin(x)}{x^2-9}\right]$

   d) $y' = e^x \sec(3x^2-9)\left[1 + 6x\tan(3x^2-9)\right]$

   e) $y' = \frac{1}{3}\left[\frac{1}{x} - \frac{5}{x+4} - \frac{2x}{x^2-9}\right]$

8) $y = -\frac{1}{8}x - \frac{13}{4}$

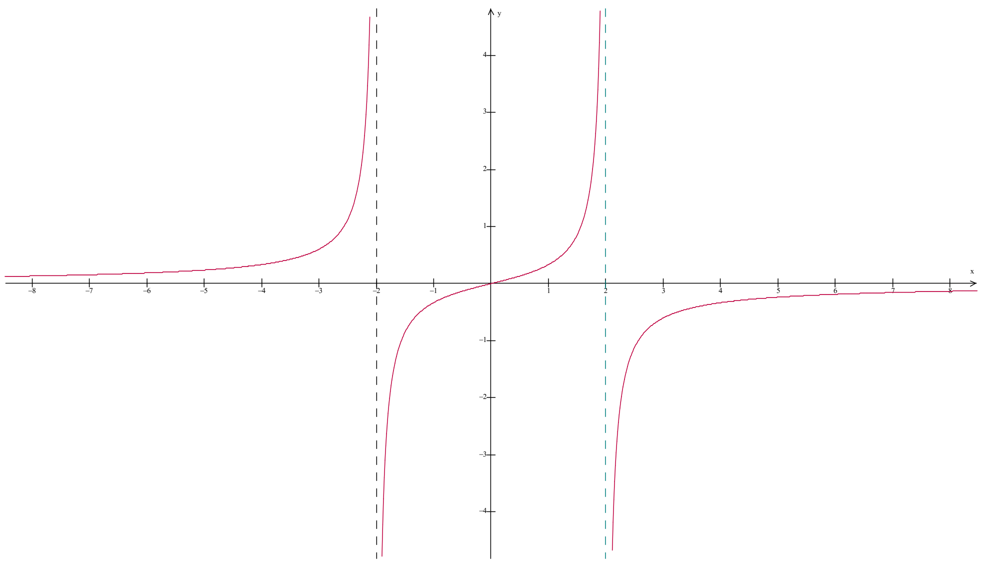
9)a) $y' = \frac{-2x - y}{x + 2y}$ b) $\left(\sqrt{3}, -\sqrt{3}\right) and \left(-\sqrt{3}, \sqrt{3}\right)$

10) $\pm\frac{1}{e^{1/3}}$ 11) $\frac{dh}{dt} = \frac{12}{\pi} cm/s$ 12) a) $(-\infty,-2)\ U\ (-2,2)\ U\ (2,\infty)$ b) (0,0)

c) ± 2 d) 0 e)f(x) increasing on $(-\infty,-2)\ U\ (-2,2)\ U\ (2,\infty)$ f)None

g) concave up on $(-\infty,-2)\ U\ (0,2)$ and concave down on $(-2,0)\ U\ (2,\infty)$ h) (0,0)

i)



13) a) $f(-1) = 3$, $f\left(\frac{125}{8}\right) = -\frac{67}{8}$ b) absolute maximum at (-1,3) , absolute minimum at $\left(\frac{125}{8}, \frac{-67}{8}\right)$

14) $r = \sqrt[3]{\frac{5}{16\pi}}$ m ; $h = \sqrt[3]{\frac{256}{25\pi}}$ m

15) $s(t) = \frac{1}{2}t^3 - 3\cos(t) + 4t + 6$

16) a) $e - 4$ b) $\frac{3\sqrt[3]{x^5}}{5} - 2\cos(x) - e^2 x + c$ c) $x - \sin(x) + c$

17) $\frac{1}{2}\left(e^3 - \frac{1}{e}\right)$ 18) $\frac{23}{4}$ 19) $3x^2 e^{x^9}$