Mathematical Models 2
Final Examination
Winter 2011
Instructor: Bob DeJean

For numeric answers, please use 4 decimal places

2 mark questions

What is the Root-Mean-Square of this current: i = 15 sin(4t) ?

Here are two currents: $i_1 = 3 + 4 \sin x + 5 \sin 2x + \ldots$

and $i_2 = 8 - 7 \sin x - 6 \sin 2x - \ldots$

What is the result of adding the currents ?

Is $y = \frac{x}{x+1}$ a solution of $x^2y' + y^2 = 0$?

Is $y = x^3 - x^2$ a solution of $xy' = x^2 + 3y$?

3 mark questions

What is the equation of the line tangent to $y = \sqrt[5]{x}$ at the point where x = 32 ?

Find the derivatives:

y = 4 sin 3x cos 2x

$$y = \frac{tan^{-1}x}{x}$$

$$y = 5x \ln 5x$$

$$y = sin^{-1}(e^x)$$

For what value of x does $y = 2x^3 - 15x^2 + 24x + 11$ reach its maximum between 0 and 6? For what value does it reach its minimum ?

4 mark questions

Use Newton's Method to solve $x^3 + 5x - 11 = 0$ accurate to 4 decimal places.

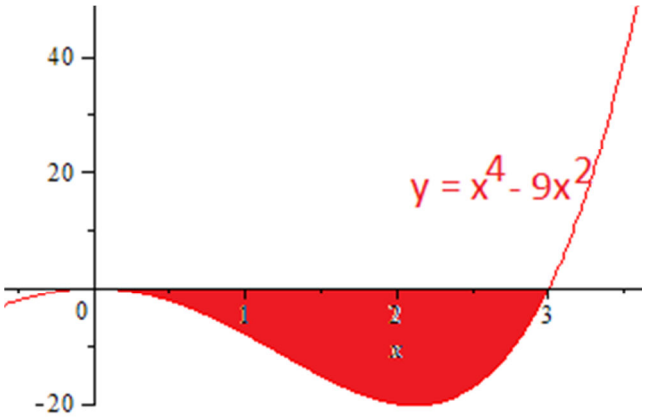
Integrate:

$$\int 6x^3 - 4x^2 + 18x \ dx =$$

$$\int \frac{16}{\sqrt{x}} dx =$$

$$\int_4^{12} \sqrt{2x+1} dx =$$

Find this area:



Use Simpson's Rule with n= 6 to approximate this integral:

$$\int_1^{2.2} \frac{6}{1+x^3}\,dx$$

The charge on a capacitor is the integral of the current going through it. Start your capacitor off with a charge of 20 Coulombs and run a current of i = 5t (i in milliamps, t in seconds) through it. What is the charge after 4 seconds ?

Find the area between y = sin x, y = cos x, x = 0 and x = 0.5 .

Integrate:

$$\int \frac{(lnx)^4}{x} dx =$$

$$\int_3^4 \frac{dx}{x-1} =$$

Integrate

$$\int 8 \sin 2x \cot 2x \, dx =$$

$$\int sin^2 x \, cos^3 x \, dx =$$

$$\int \frac{2x}{x^2 + 16} dx =$$

$$\int x^2 \ln(3x) \, dx =$$

Find the solution of $xy\,y' = 4$

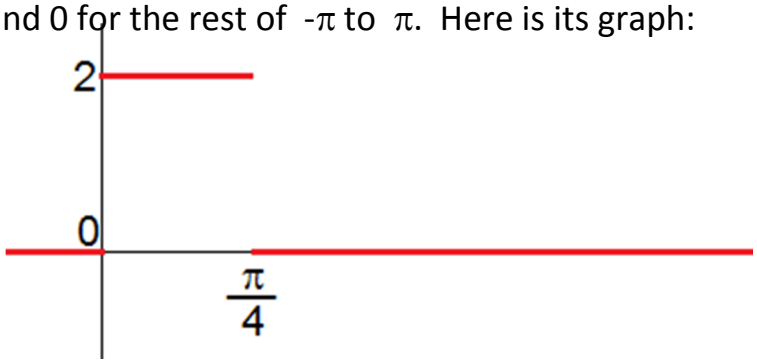
Find the solution of $xy' - 3y = 9x^5$

Find the solution of $y' + y = x\,e^{-x}$

6 mark question

Consider the function that is 2 for $0 \le x \le \frac{\pi}{4}$

and 0 for the rest of $-\pi$ to π. Here is its graph:



I am interested in its Fourier Expansion.

What is a_0 =

What is a_1 =

What is b_1 =

Use these values to write the beginning of the Fourier Expansion of the function.

Backup

Find the equation of the line normal to $y = \frac{x}{x-4}$ at the point (6, 3).

$y = \frac{secx}{x}$

$y = 7 \sin^{-1}(2x - 15)$

$y = \ln(8x^2)$

???? $y = ln\left(\frac{6x-13}{\sqrt{x}}\right)$

$y = 16x^3e^{5x}$

Find the derivative implicitly: $x^2 - y^4 = x \sin y$

Bryan wants to fence an area of 1250 m^2 for his employees parking. To keep it simple, the parking lot will be a rectangle, with fence on 3 sides. The fourth side is along the side wall of the Les Entreprises Bryan building. What dimensions (length and width) use the minimum amount of fence ?

What is the Differential of $y = e^x - 3^x$?

$\int (3x - 5)^7 \, dx =$

$\int_1^3 \frac{dx}{x^2} =$

Find the area between $y = 2^x$, $x = 1$, $x = 2$ and the x-axis. A diagram might be useful.

Use the Trapezoidal Rule with n = 5 to approximate this area:

$\int_4^9 lnx + 1dx$

What is the area between the two curves: $y = x^3$ and $y = 9x^2 - 24x$?

$$\int \sin x \cos x \, dx =$$

$$\int \frac{sec2x \; tan2x}{1 + sec2x} dx =$$

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx =$$

$$\int \frac{dx}{\sqrt{9 - x^2}} =$$

$$\int 3x \; cos4x \; dx =$$

$$\int sin(3x) \, e^{cos(3x)} \; dx =$$

Find the solution of sin y y' – 4x = 0

Answers
10.6066
11 – 3 sin x – sin 2x …
no
yes

y = 1/80 x + 1.6
y’ = 12 cos 3x cos 2x – 8 sin 3x sin 2x
$y' = \frac{x - (1 + x^2)tan^{-1}x}{x^2(1 + x^2)}$
y’ = 5 ln 5x + 5
$y' = \frac{e^x}{\sqrt{1 - e^{2x}}}$
Max at x = 6
Min at x = 4

1.5106
$1.5x^4 – 4/3\ x^3 + 9x^2 + C$
$32\sqrt{x} + C$
32.6667

32.4
1.6434

This should be 20.040 coulombs, because milliamps changes the units to millicoulombs.
0.3570
$\frac{1}{5}(lnx)^5 + C$
0.4054

4 sin 2x + C
$\frac{1}{3}sin^3x - \frac{1}{5}sin^5x$+ C
$\ln|x^2 + 16| + C$
$\frac{1}{3}x^3\ln(3x) - \frac{1}{9}x^3 + C$

$$y = \pm\sqrt{8\ln|x| + C}$$
$$y = \frac{9}{2x} + \frac{C}{x^3}$$
$$y = \frac{9}{2x} + \frac{C}{x^3}$$
$$y = e^{-x}\left(\frac{x^2}{2} + C\right)$$

$a_0 = 0.25$
$a_1 = 0.4505$
$b_1 = 0.1865$
f = 0.25 + 0.4505 cos x + … + 0.1865
sin x + …