(Marks)

1. Evaluate each of the following integrals without the use of integration tables.

(3) (a) $\displaystyle\int \left(3x^5 - \frac{3}{x-4} + 5^x - \frac{4}{\sec x} + \frac{1}{\pi} + \frac{3}{\sqrt{x}}\right) dx$

(3) (b) $\displaystyle\int \frac{3x^3 + 12x^2 + 5x - 9}{x+2}\, dx$

(3) (c) $\displaystyle\int \frac{\sin(\ln x)}{x}\, dx$

(4) (d) $\displaystyle\int_{1/2}^{1} \frac{1}{x^2}\left(1 + \frac{1}{x}\right)^3 dx$

(4) (e) $\displaystyle\int (x^2+1)\cos(x)\, dx$

(4) (f) $\displaystyle\int_0^{\pi/8} \sec^2(2x)\, e^{\tan(2x)}\, dx$

(4) (g) $\displaystyle\int \frac{x^2}{\sqrt{x+2}}\, dx$

(4) (h) $\displaystyle\int \frac{7x^2 - 3x - 8}{(x-1)^2(x+3)}\, dx$

(4) 2. Use Simpson's rule with $n = 6$ to approximate $\displaystyle\int_1^{13} \frac{2}{\sqrt{x + \ln x}}\, dx$.
Round your answer to four decimal places.

3. Use the table of integrals to solve each of the following.
In each case, state the formula number and justify its use.

(4) (a) $\displaystyle\int \frac{2\, dx}{(x-1)^2\sqrt{7 + 2x - x^2}}$

(4) (b) $\displaystyle\int x\sqrt{x^4 - 9}\, dx$

(2) 4. Determine if the function $y = x^4 + x + 4\ln(x)$ is a solution of the differential equation $xy'' + y' = 16x^3$

5. Solve the following differential equations for y:

(4) (a) $y' = \dfrac{4x\sin(x^2)}{e^y}$ with condition $y(0) = 0$

(4) (b) $y' = 6x^2\sqrt{y+8}$ with condition $y(0) = 1$

(4) 6. ROAM inc. has upgraded its computer infrastructure by purchasing several new computers for a total value of \$8 000. The rate of the depreciation value V in dollars at time t in years is proportional to the square of its value V. The computers will be worth \$5 000 three years later.
What is the value of the computers after 5 years.

(4) 7. Given the curves $f(x) = x^2 - 3x + 2$ and $g(x) = -x^2 + 9x - 8$, determine

(a) the point(s) of intersection of $f(x)$ and $g(x)$,

(b) sketch and find the area of the region bounded by $f(x)$ and $g(x)$.

(6) 8. Given the demand function $p = \dfrac{255}{x+3}$ and the supply function $p = x + 5$,

(a) find the equilibrium point,

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(b) sketch and identify the regions representing the consumer and producer surpluses,

(c) evaluate the consumer's surplus.

9. Use l'Hôpital's rule to evaluate the following limits

(3) (a) $\lim_{x \to 0} \dfrac{e^x - 1 - x}{x^2}$

(3) (b) $\lim_{x \to \infty} \dfrac{x+3}{\ln x}$

10. Evaluate each improper integral and state whether it converges or diverges

(4) (a) $\displaystyle\int_1^\infty \frac{4}{\sqrt{(3x+1)^3}}\,dx$

(4) (b) $\displaystyle\int_{-2}^0 \frac{x^2}{x^3+8}\,dx$

(3) 11. Consider the sequence $\left\{\dfrac{27}{15}, -\dfrac{81}{24}, \dfrac{243}{33}, -\dfrac{729}{42}, \ldots\right\}$

(a) Give the next term of the sequence.

(b) Give an expression for the n^{th} term of the sequence.

12. Determine the convergence or divergence of each sequence $\{a_n\}$.
If the sequence converges, find the limit.

(3) (a) $a_n = \dfrac{n!}{5n}$

(3) (b) $a_n = \dfrac{\sqrt{9n^4+4}}{3n^2-1}$

13. Determine with *justification* if each of the following series is convergent or divergent.
If the series is convergent, *find its sum*.

(3) (a) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{2n}+3}{e^n+1}$

(3) (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n-1}}{4^{n+2}}$

(3) 14. Given the number $6.\overline{42}$, express it using a geometric series, find the sum of the geometric series and write the number as the ratio of two integers.

(3) 15. A deposit of \$200 is made at the beginning of each month for 10 years into an account that pays an annual rate of 3% compounded monthly. Find the total amount in this account at the end of 10 years. Give the answer to 2 decimals.

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ANSWERS

(1 a) $\frac{1}{2}x^6 - 3\ln|x-4| + \dfrac{5^x}{\ln(5)} - 4\sin x + \dfrac{1}{\pi}x + 6\sqrt{x} + C$; (1 b) $x^3 + 3x^2 - 7x + 5\ln|x+2| + C$

(1 c) $-\cos(\ln x) + C$; (1 d) $\dfrac{65}{4} = 16.25$; (1 e) $(x^2+1)\sin(x) + 2x\cos(x) - 2\sin(x) + C$

(1 f) $\frac{1}{2}(e-1) \approx 0.86$

(1 g) $\frac{2}{5}(x+2)^{5/2} - \frac{8}{3}(x+2)^{3/2} + 8(x+2)^{1/2} + C = 2x^2\sqrt{x+2} - \frac{8}{3}x(x+2)^{3/2} + \frac{16}{15}(x+2)^{5/2} + C$

(1 h) $3\ln|x-1| + 4\ln|x+3| + \dfrac{1}{x-1} + C$; (2) 9.3800

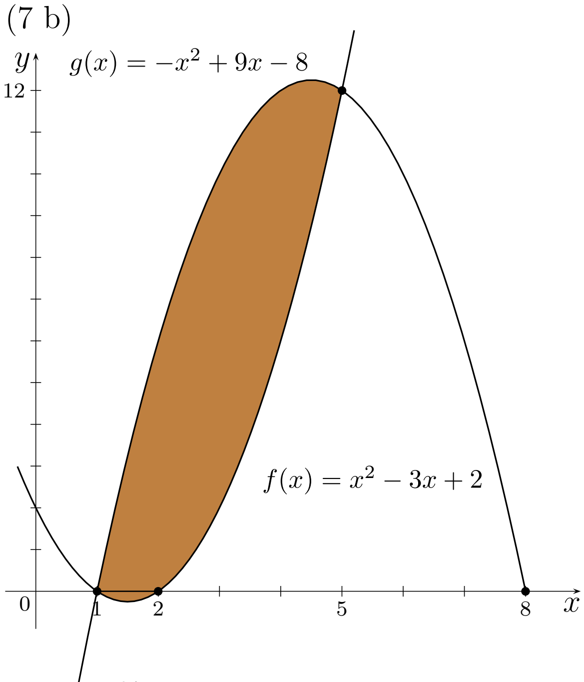
(3 a) complete the square then F20: $\dfrac{-\sqrt{7+2x-x^2}}{4(x-1)} + C$

(3 b) substitution then F22: $\frac{1}{4}x^2\sqrt{x^4-9} - \frac{9}{4}\ln|x^2+\sqrt{x^4-9}| + C$; (4) it is not a solution

(5 a) $y = \ln\left(3 - 2\cos(x^2)\right)$; (5 b) $y = (x^3+3)^2 - 8 = x^6 + 6x^3 + 1$; (6) $V = \$4000$

(7 a) points of intersection:
$(1,0)$; $(5,12)$

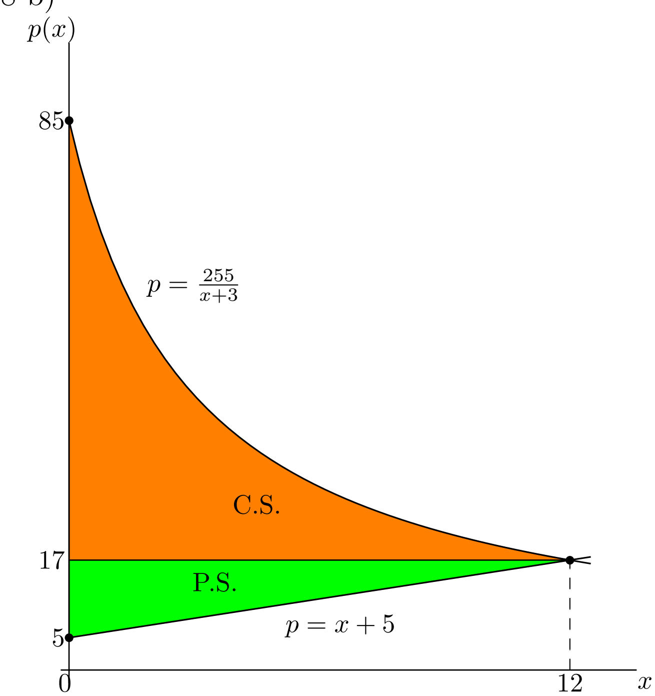
(7 b)



Area $= \frac{64}{3} \approx 21.3$ square units

(8 a) point of equilibrium: $(12, 17)$

(8 b)



(8 c) C.S.=\$ 206.41

(9 a) $\frac{1}{2}$; (9 b) ∞ ; (10 a) converges to $\frac{4}{3}$; (10 b) divergent

(11 a) $a_5 = \dfrac{2187}{51}$; (11 b) $a_n = (-1)^{n+1}\dfrac{3^{n+2}}{9n+6}$

(12 a) divergent ; (12 b) converges to 1 ; (13 a) divergent ; (13 b) converges to $\frac{1}{16}$

(14) $\dfrac{212}{33}$; (15) \$ 28018.15