(Marks)

(4) 1. Use the graph to find the following limits. Use ∞, $-\infty$, or DNE where appropriate.

(a) $\lim_{x \to -2} f(x) =$ ______________

(b) $\lim_{x \to -\infty} f(x) =$ ______________

(c) $\lim_{x \to \infty} f(x) =$ ______________

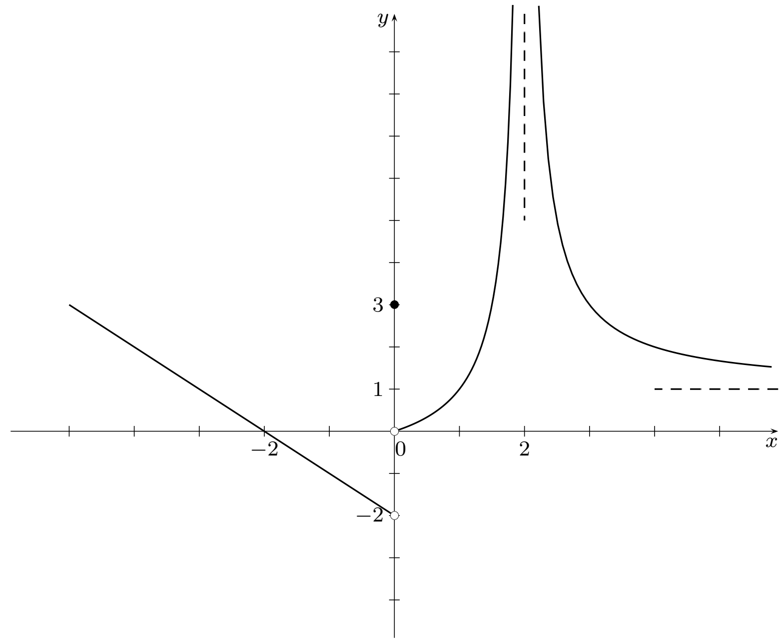
(d) $\lim_{x \to 0^-} f(x) =$ ______________

(e) $\lim_{x \to 2} f(x) =$ ______________

(f) $\lim_{x \to 0^+} f(x) =$ ______________

(g) $f(0) =$ ______________

(h) $f(2) =$ ______________



(15) 2. Use algebraic techniques to evaluate the following limits. Identify the limits that do not exist, and use ∞ or $-\infty$ where appropriate. **Show your work**.

(a) $\lim_{x \to -2} \dfrac{2x^2 + x - 6}{3x^2 - 12}$

(b) $\lim_{x \to 7} \dfrac{2 - \sqrt{x-3}}{x - 7}$

(c) $\lim_{x \to -\infty} \dfrac{4x^4 - 3x^3 - 4}{-2x^5 + 1 - 5x}$

(d) $\lim_{x \to 0^-} \left(\dfrac{1}{x} - \dfrac{1}{x^2} \right)$

(e) $\lim_{x \to 3^+} f(x)$ where $f(x) = \begin{cases} x^2 + 3 & \text{for } x < 3 \\ \dfrac{x-3}{x^2-9} & \text{for } x > 3 \end{cases}$

(3) 3. Use the definition of continuity to find the value(s) of x for which the following function is discontinuous.

$$f(x) = \begin{cases} x^2 - 3 & \text{for } x < -2 \\ \dfrac{1}{(2x+7)(x-4)} & \text{for } x \geq -2 \end{cases}$$

(3) 4. Find the value(s) of k such that $f(x)$ is continuous for all real numbers

(Marks)

$$f(x) = \begin{cases} -x^2 - 5k & \text{for} \quad x < 2 \\ k^2 - \dfrac{20}{x} & \text{for} \quad x \geq 2 \end{cases}$$

(5) 5. (a) Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \sqrt{3x-8}$.

(b) Check your answer using the derivative rules.

(c) Use your answer in part a) to find the slope of the line tangent to $f(x)$ at $x = 4$.

(28) 6. Find $\dfrac{dy}{dx}$ for each of the following. **Do not simplify your answers**.

(a) $y = \dfrac{4}{\sqrt{x}} - \sqrt[4]{x} + x^4 - 4^x$

(b) $y = 5x\,\log_2(\sin x)$

(c) $y = \sqrt{\dfrac{e^{3x+2}}{\sec 3x}}$

(d) $y = \ln\Big(\big((3x-2)^5\,(4-2x)^6\big)^2\Big)$

(e) $y = x^3\cos^2 x + x^3\sin^2 x + \pi$

(f) $y = 5\,(3x)^{e^x}$

(g) $4x^2y^3 + x^3 = (3x+y)^2$

(4) 7. Determine the $x-$value(s) where $f(x)$ has horizontal tangents given $f(x) = \dfrac{3}{4x^2+7x-2}$

(4) 8. Given the function $f(x) = e^{3x}\cos(1+x)$, determine $f''(0)$.

(4) 9. Use the second derivative test to determine the relative extrema of $f(x) = 3x^3 - 9x$

(4) 10. Determine the absolute maximum and minimum of $f(x) = x^2\,e^{-x}$ on the interval $[-1, 1]$.

(10) 11. Given $f(x) = \dfrac{3x^2}{x-1}$; $f'(x) = \dfrac{3x(x-2)}{(x-1)^2}$; $f''(x) = \dfrac{6}{(x-1)^3}$

(a) Find the $y-$intercept, $x-$intercept, any vertical and horizontal asymptotes, relative extrema and points of inflection (if any).
Find the intervals where f is increasing, decreasing, concave up and concave down.

(b) Sketch a graph of $f(x)$.

(Marks)

(5) 12. For brunch parties, a catering company charges \$8 per person for groups of 50 people or fewer. In order to encourage large groups, for *each* additional person above fifty, the caterer will reduce the price for *everyone* by \$0.05.

(a) What size group will produce maximum revenue for the caterer?

(b) What is the maximum revenue?

(6) 13. A storage box with square base and no top is to have a volume of 40 m^3. Material for the base costs \$5 per square meter. Material for the sides costs \$4 per square meter. Find the cost of materials for the cheapest such container. Use a test to verify that a minimum was found.

(5) 14. The demand equation for a product is $p = 1200 - 40\sqrt{x}$

(a) Find the elasticity of demand at $x = 300$.

(b) Is the demand elastic or inelastic when $x = 300$? In your own words, describe what this represents.

(c) Does the demand have unit elasticity at $x = 400$? Justify your answer.

(Marks)

Answers

1. a) 0 b) $+\infty$ c) 1 d) -2 e) $+\infty$ f) 0 g) 3 h) undefined
2. a) $\frac{7}{12}$ b) $-\frac{1}{4}$ c) 0 d) $-\infty$ e) $\frac{1}{6}$
3. $x = -2$ or $x = 4$ 4. $k = -6$; $k = 1$
5. a) Use $f\,'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ b) $f\,'(x) = \dfrac{3}{2\sqrt{3x-8}}$ c) $f'(4) = \frac{3}{4}$
6. a) $\dfrac{dy}{dx} = -2x^{-3/2} - \frac{1}{4}x^{-3/4} + 4x^3 - 4^x\ \ln(4)$ b) $\dfrac{dy}{dx} = 5\ \log_2(\sin x) + 5x\,.\,\dfrac{\cos x}{\sin x\ \ln(2)}$

c) $\dfrac{dy}{dx} = \frac{1}{2}\left(\frac{e^{3x+2}}{\sec 3x}\right)^{-1/2}\left[\frac{3e^{3x+2}\sec 3x - 3\ \sec 3x \tan 3x\, e^{3x+2}}{\sec^2 3x}\right]$ d) $\dfrac{dy}{dx} = \dfrac{30}{3x-2} + \dfrac{-24}{4-2x}$

e) $\dfrac{dy}{dx} = 3x^2\cos^2 x - 2\sin x\cos x\,.\,x^3 + 3x^2\sin^2 x + 2\sin x\cos x\,.\,x^3$ f) $\dfrac{dy}{dx} = 5\,(3x)^{e^x}\left[e^x\ln(3x) + \dfrac{e^x}{x}\right]$

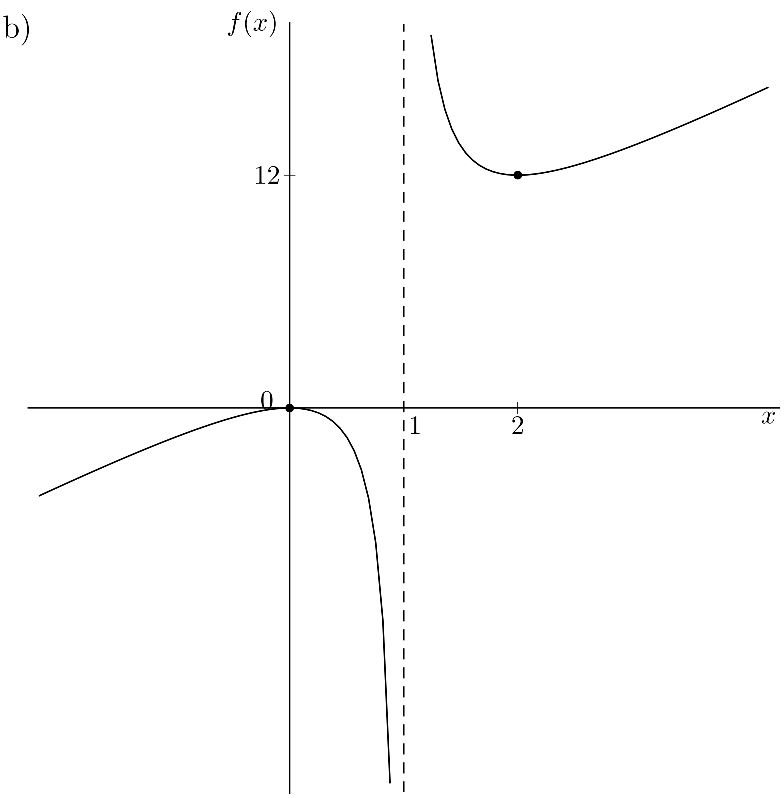
g) $\dfrac{dy}{dx} = \dfrac{18x + 6y - 8xy^3 - 3x^2}{12x^2y^2 - 6x - 2y}$ 7. $x = -\frac{7}{8}$ 8. $8\cos(1) - 6\sin(1) \approx -0.73$

9. relative maximum at $(-1, 6)$ and relative minimum at $(1, -6)$

10. absolute maximum is 2.72 at $x = -1$ and absolute minimum is 0 at $x = 0$

11. a) y-int and x-int:(0,0)
vertical asymptote: $x = 1$
horizontal asymptote: none
relative maximum: $(0, 0)$
relative minimum: $(2, 12)$
PI: none
Inc: $(-\infty, 0) \cup (2, +\infty)$
Dec: $(0, 1) \cup (1, 2)$
CU: $(1, +\infty)$
CD: $(-\infty, 1)$

11 b)



12. a) 105 people
12. b) maximum revenue is \$551.25

13. cost of materials: \$240

14. a) $\eta(300) = -1.46$
b) elastic at $x = 300$ units;
if price decreases by 10%,
the quantity increases by 14.6%;
revenue will increase
c) yes, unit elasticity at $x = 400$ units