1. Differentiate with respect to x and simplify your answer: (3)

$$y = \sin^{-1}\left(\sqrt{1-x^2}\right) + \cot^{-1}\left(\frac{1}{x}\right) - \sec^{-1}(5), \quad 0 < x \le 1$$

2. Evaluate the following limits: (9)

a) $\lim\limits_{x \to 1^+}\left(\dfrac{x}{x-1} - \dfrac{1}{\ln x}\right)$

b) $\lim\limits_{x \to 0^+}\dfrac{3x}{\arctan 3x}$

c) $\lim\limits_{x \to \frac{\pi}{2}^-}(\tan x)^{\cos x}$

3. Evaluate the following integrals:

a) $\displaystyle\int \frac{(x+1)}{\sqrt{2x+1}}\,dx$ (4)

b) $\displaystyle\int_0^{\frac{\pi}{4}} (1+\tan x)^3 \sec^2 x\,dx$ (4)

c) $\displaystyle\int x^{5/2} \ln x\,dx$ (4)

d) $\displaystyle\int \frac{\sec^4 x}{\tan^2 x}\,dx$ (4)

e) $\displaystyle\int \frac{x^2+x}{x^4-1}\,dx$ (4)

f) $\displaystyle\int \frac{x^2}{\sqrt{1-4x^2}}\,dx$ (4)

4. Determine whether the following improper integrals converge or diverge. If an integral converges, find its exact value.

a) $\displaystyle\int_0^9 \frac{1}{\sqrt[3]{1-x}}\,dx$ (4)

b) $\int_{-\infty}^{0} \frac{1}{1+4x^2}\,dx$ (3)

5. Solve the differential equation. Provide a solution in a) implicit form and b) explicit form. (4)

$$y'\sqrt{1-x^2} = xy; \quad y(0)=1,\ y>0$$

6. Compute the exact area of the region bounded by $y = x$ and $y = \ln x$ on the interval $\left[1, e^2\right]$ (3)

7. Let R be the region bounded by $x = -y^2 - 2y$ and the y-axis. Set up, **but DO NOT EVALUATE ,** (6)

integrals corresponding to the volume of the solid obtained by rotating R around

a) the $x-$axis b) the $y-$ axis c) the line $y = 5$

8. Determine whether the sequence converges or diverges. If it converges, to what does it converge ? (3)

a) $\left\{\frac{3^n}{n!}\right\}$

b) $\left\{n 2^{1/n}\right\}$

c) $\left\{\frac{1+\cos n}{\sqrt{n}}\right\}$

9. Let $\sum_{n=1}^{\infty} a_n$ be a series whose n^{th} partial sum is given by $S_n = \frac{2n+1}{3n^2 - n}$ (3)

a) Determine whether the series converges or diverges. If it converges, find its sum.

b) Find a_3

10. Provide an example of each of the following: (3)

a) A sequence $\{a_n\}$ such that $\lim_{n\to\infty} a_n = 0$, but $\sum_{n=1}^{\infty} a_n$ diverges

b) A series $\sum_{n=1}^{\infty}(a_n + b_n)$ which converges, but neither $\sum_{n=1}^{\infty} a_n$ nor $\sum_{n=1}^{\infty} b_n$ converges.

c) function f such that $\lim_{x\to\infty} f(x)$ does not exist, but $\lim_{n\to\infty} f(n)$ exists.

11. Suppose f and g are both decreasing functions such that $0 < f(x) < g(x)$ for all $x \geq 1$ and that (5)

$\sum_{n=9}^{\infty} f(n)$ converges. Fill in each blank with the appropriate word (*must, might, cannot*)

a) the series $\sum_{n=1}^{\infty} f(n)$ ____________________ converge

b) the integral $\int_1^{\infty} f(x)dx$ ____________________ converge

c) the series $\sum_{n=9}^{\infty} g(n)$ ____________________ converge

d) the series $\sum_{n=9}^{\infty} \frac{1}{f(n)}$ ____________________ converge

e) if $\lim_{n\to\infty} \frac{g(n)}{f(n)} = 5^8$, then $\sum_{n=9}^{\infty} g(n)$ ______________ converge

12. Determine whether the series converges or diverges. Justify your answers.

a) $\sum_{n=1}^{\infty} \frac{\arctan(n)}{\sqrt{n+1}}$ (3)

b) $\sum_{n=1}^{\infty} \frac{n^2+1}{(n+1)^2} \cos\left(\frac{\pi}{4n}\right)$ (3)

c) $\sum_{n=1}^{\infty} \left(\frac{3n}{n^3+1} - \frac{2^{n-1}}{3^n}\right)$ (3)

13. Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n n^2}{(2n)!}$ (3)

b) $\sum_{n=1}^{\infty} \frac{\cos[(n+1)\pi]}{n^{1/3}}$ (2)

c) $\sum_{n=1}^{\infty} (-1)^n 4^n \left(2 - \frac{1}{n}\right)^{2n}$ (2)

continued

d) $\sum_{n=1}^{\infty} \frac{\sin(2^n)}{(n+1)(n+2)}$

14. Find the radius and interval of convergence for the power series $\sum_{n=1}^{\infty} (-1)^n \frac{(x-2)^n}{n2^n}$ **(4)**

15. Write the Maclaurin series for $f(x) = e^{-2x}$ in sigma notation with a formula for the n^{th} term. **(2)**

16. a) Write the first 5 terms of the Taylor series for $f(x) = (2x-1)^{-2}$ with center 1. **(6)**

b) Write the series in sigma notation with a formula for the n^{th} term.

Answers

1. $\frac{1}{x^2+1} - \frac{1}{\sqrt{1-x^2}}$ **2.** $\frac{1}{2}$, 1, 1 **3.** $\sqrt{2x+1}(x-1)+C$, $\frac{15}{4}$, $\frac{2}{7}x^{7/2}\left(\ln x - \frac{2}{7}\right)+C$

$-\cot x + \tan x + C$, $\frac{1}{2}\left[\ln|x-1| - \frac{1}{2}\ln(x^2+1) + \tan^{-1} x\right] + C$, $\frac{1}{16}\sin^{-1}(2x) - \frac{1}{8}x\sqrt{1-4x^2} + C$

4. $-\frac{9}{2}$, $\frac{\pi}{4}$, **5.** $\ln y = 1 - \sqrt{1-x^2}$, $y = e^{1-\sqrt{1-x^2}}$ **6.** $\frac{1}{2}e^4 - e^2 - \frac{3}{2}$

7. Shell: $\int_{-2}^{0} -2\pi y(y^2+2y)\,dy$ **Ring:** $\int_{-2}^{0} \pi(y^2+2y)^2\,dy$ **Shell:** $\int_{-2}^{0} 2\pi(y-5)(y^2+2y)\,dy$

8. C to 0 by the Sq. theorem, D, C to 0 by the Sq. theorem

9. C to 0, its sum is zero, $a_3 = -\frac{5}{24}$ **10.** $\sum \frac{1}{n^p}, 0 < p \le 1$, $\sum \frac{1}{n}$ *and* $\sum -\frac{1}{n}$, $f(n) = \cos 2n\pi$

11. m ust, must, might, cannot, must

12. D by LCT with $\sum_{1}^{\infty} \frac{1}{\sqrt{n+1}}$ which D, D by NTT for D, C: sum of covergent series

13. AC by the Ratio Test, Not AC, but C by the AST, hence CC, D by the NTT for D **OR** D by the Ratio Test

AC by the Comparison Test of $\sum_{1}^{\infty} |a_n|$ with $\sum \frac{1}{(n+1)(n+2)}$ which C

14. Rad. is 2, Interval of C is $(0, 4]$ **15.** $e^{-2x} = \sum_{0}^{\infty} \frac{(-1)^n 2^n x^n}{n!}$ **16.** $(2x-1)^{-2} = \sum_{0}^{\infty} (-1)^n (n+1) 2^n (x-1)^n$