(Marks)

(2) 1. Some democracies such as Australia use a preferential voting system where voters rank the candidates in order of preference. Suppose there are seven candidates running for political office and each voter must mark his or her first, second and third choices on the ballot. How many different ballots could a voter possibly cast?

(2) 2. List the elements of the set $\{x \in \mathbb{Z} : x^2 + 1 \leq 10\}$.

(5) 3. Suppose that the set S has 10 elements.

(a) How many proper subsets does S have?

(b) How many subsets of S have exactly five elements?

(c) How many subsets of S have two or more elements?

(5) 4. Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the universal set, let $A = \{1, 2, 3, 5, 8\}$, $B = \{0, 2, 4, 6, 8\}$ and $C = \{0, 1, 4, 9\}$. Find:

(a) $A \cup (B \cap C)$

(b) $\overline{A} \cup B$

(c) $(\overline{A \cap B}) \cap (A - C)$

(4) 5. Draw Venn diagrams and show by hatching the following sets. What can you conclude?

(a) $\overline{A} \cap (B \cup C)$

(b) $(\overline{A} \cap B) \cup (\overline{A} \cap C)$

(6) 6. For each expression, name the property from the given list and say whether it is a set property, a network property, or a logic property: *associativity, commutativity, distributivity, identity, idempotent, de Morgan, closure, complement, property of 1 (or 0), tautology, contradiction.*

a) $p \wedge (q \wedge r) \leftrightarrow (p \wedge q) \wedge r$ ____________________

b) $A \cup \overline{A} = U$ ____________________

c) $A + A = A$ ____________________

d) $A + B \cdot C = (A + B) \cdot (A + C)$ ____________________

e) $\overline{A \cap B} = \overline{A} \cup \overline{B}$ ____________________

f) $(p \vee \sim p) \leftrightarrow t$ ____________________

(6) 7. Provide truth tables for the following propositions and identify any which are tautologies or contradictions.

(a) $\sim (p \wedge q) \ \vee (\sim (p \rightarrow \sim q))$

(b) $(p \rightarrow (q \vee r)) \vee (q \wedge p)$

(6) 8. Use a truth table to determine whether the argument is valid or not.

H: Either Shakira is the best musician on the planet or monkeys can fly.
If monkeys can fly then elephants can swim.

C: Either Shakira is the best musician on the planet or elephants can swim.

(3) 9. Use a Venn diagram to determine the validity of the argument.

(Marks)

H: All spiders have eight legs.
Some creatures that lay eggs have eight legs.

C: Some spiders lay eggs.

(3) 10. Write the inverse, converse and contrapositive of the statement: "If I get the job, then I will move to Moscow." Clearly identify each statement. Are any of them logically equivalent to the original?

(4) 11. Provide a Boolean table for the following expression: $(A+\overline{BC})\cdot(\overline{A+C})$

(2) 12. Draw a network corresponding to the Boolean expression $(A+BC)\cdot(\overline{A}+\overline{C})+\overline{B}D$.

(5) 13. Simplify each expression, justifying each step using properties of Boolean algebra.

(a) $A+\overline{A}B+\overline{B}C$

(b) $A\overline{B}C+\overline{A}C(AB+\overline{CB})$

(2) 14. Solve the system using the substitution method.

$$\begin{cases} x-3y &= 5 \\ 3x-2y &= 1 \end{cases}$$

(2) 15. Solve the system using the addition/multiplication method.

$$\begin{cases} 3x-4y &= -6 \\ 2x+y &= 7 \end{cases}$$

(2) 16. For which values of k is the system $\begin{cases} x-5y &= 2 \\ 2x-ky &= -4 \end{cases}$ (a) dependent? (b) consistent?

(12) 17. Given: $A=\begin{bmatrix} 1 & 4 \\ -2 & 3 \\ 1 & -2 \end{bmatrix}$ $B=\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ $C=\begin{bmatrix} 4 & 3 & -3 \\ 6 & -1 & 1 \\ -4 & 0 & 0 \end{bmatrix}$ $D=\begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$

$E=\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$ $I=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

find each of the following, if possible. *If an operation is not possible, explain why.*

a) $C+2I$ (2)
b) CA (2)
c) D^{-1} (2)
d) AC (1)
e) E^{-1} (1)
f) $2A^T+3B$ (2)
g) D^2 (2)

(7) 18. Given $A=\begin{bmatrix} -3 & -6 & -11 \\ 2 & 5 & 10 \\ 1 & 2 & 4 \end{bmatrix}$ find A^{-1} using elementary row operations and verify your answer.

(3) 19. Given $A=\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 0 \\ 2 & -3 & 1 \end{bmatrix}$, explain why A^{-1} does not exist.

(Marks)

(3) 20. Given the linear system $\begin{cases} x + 2y + 3z = 5 \\ 2x + 5y + 3z = 3 \\ x + 8z = 17 \end{cases}$

a) write the system in matrix form $AX = B$;

b) if $A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$, solve the system using A^{-1}.

(6) 21. Solve the following system by Gaussian or Gauss-Jordan Elimination.

$$\begin{aligned} x + y + z &= 3 \\ x - y - 2z &= -5 \\ 2x + 3y + z &= 5 \end{aligned}$$

(3) 22. Use Gaussian or Gauss-Jordan Elimination to show the following system has no solution.

$$\begin{aligned} x + 2y + 3z &= -1 \\ -2x + y &= 3 \\ 3x + y + 3z &= 2 \end{aligned}$$

(2) 23. Suppose that the augmented matrix of a linear system has been reduced to the following reduced row echelon form $\left[\begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$.

(a) What is the general solution of this system?
(b) How many solutions does this system have?

(2) 24. Fill in the blanks with the appropriate word. In each case the appropriate word is either *must*, *might*, or *cannot*.

(a) If A is a matrix such that A^2 is invertible, then A ____________ also be invertible.

(b) A system of equations with four equations and three variables ____________ have exactly four solutions.

(c) For a matrix A, the matrices AA^T and A^TA ____________ be of the same size.

(d) The concluding statement of a valid argument ____________ be true.

(3) 25. Prove by mathematical induction that for all positive integers n

$$3 + 9 + 27 + \cdots + 3^n = \frac{3(3^n - 1)}{2}$$