Answers to Math 803-Final Exam (December 2010)

1. ${}_7P_3 = \frac{7!}{4!} = 210$

2. $\{-3, -2, -1, 0, 1, 2, 3\}$

3. (a) $2^{10} - 1 = 1023$
(b) ${}_{10}C_5 = \frac{10!}{5!5!} = 252$
(c) $\emptyset$ has zero elements, 10 subsets have one element each, so 11 subsets have zero or one element. This means that $1024 - 11 = 1013$ have 2 or more elements.

4. (a) $A \cup (B \cap C) = \{0, 1, 2, 3, 4, 5, 8\}$
(b) $\overline{A} \cup B = \{0, 2, 4, 6, 7, 8, 9\}$
(c) $\overline{(A \cap B)} \cap (A - C) = \{3, 5\}$

5. The regions in the Venn diagram includes B and C excluding A (i.e. both are equal to $(B \cup C) - A$). Both set relations are equal. This equality represents the distributive property.

6. (a) associative (logic)
(b) complement (set)
(c) idempotent (network)
(d) distributive (network)
(e) De Morgan (set)
(f) tautology (logic)

7. (a) This statement is a tautology
(b) This statement is true in all cases except for the case that p is true and q and r are false. Therefore, it is neither a tautology nor a contradiction.

8. Use a Truth table to show that the statement $(p \underline{\vee} q) \wedge (q \to r) \to (p \underline{\vee} r)$ is not a tautology (The statement is false when p and r are true and q is false and it is true otherwise, so it is not a tautology.). The argument is invalid.

9. Invalid argument. Let S, L and E represent the set of all spiders, creatures with 8 legs and creatures that lay eggs respectively. The relation between S and E is then unclear.

10. Inverse: If I don't get the job, then I will not move to Moscow.

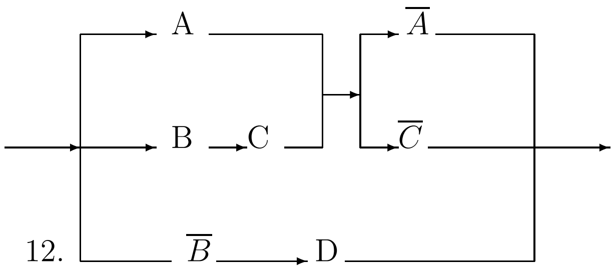
Converse: If I move to Moscow, then I get the job.

Contrapositive: If I don't move to Moscow, then I won't get the job.

Contrapositive is equivalent to the original.

11.

A	B	C	BC	$\overline{BC}$	$A+\overline{BC}$	$A+C$	$\overline{A+C}$	$(A+\overline{BC})\overline{(A+C)}$
1	1	1	1	0	1	1	0	0
1	1	0	0	1	1	1	0	0
1	0	1	0	1	1	1	0	0
1	0	0	0	1	1	1	0	0
0	1	1	1	0	0	1	0	0
0	1	0	0	1	1	0	1	1
0	0	1	0	1	1	1	0	0
0	0	0	0	1	1	0	1	1

12. 

13. (a)

$$\begin{aligned} A+\overline{A}B+\overline{B}C &= (A+\overline{A})(A+B)+\overline{B}C \\ &= 1(A+B)+\overline{B}C \\ &= A+B+\overline{B}C \\ &= A+(B+\overline{B})(B+C) \\ &= A+1(B+C) \\ &= A+B+C \end{aligned}$$

(b)

$$\begin{aligned} A\overline{B}C+\overline{A}C(AB+\overline{CB}) &= A\overline{B}C+\overline{A}C(AB+\overline{C}+\overline{B}) \\ &= A\overline{B}C+(\overline{A}A)(CB)+\overline{A}(C\overline{C})+\overline{A}C\overline{B} \\ &= A\overline{B}C+0+0+\overline{A}\,\overline{B}C \\ &= A\overline{B}C+\overline{A}\,\overline{B}C \\ &= (A+\overline{A})(\overline{B}C) \\ &= 1(\overline{B}C) \\ &= \overline{B}C \end{aligned}$$

14. Substitute $x = 3y + 5$ in the second equation to get $y = -2$. This would give $x = -1$

15. Multiply, e.g., the second equation by 4 and add them up to get $x = 2$; it then gives $y = 3$.

16. (a) Never

(b) $k \neq 10$

17. (a) $\begin{bmatrix} 6 & 3 & -3 \\ 6 & 1 & 1 \\ -4 & 0 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} -5 & 31 \\ 9 & 19 \\ -4 & -16 \end{bmatrix}$

(c) $\frac{1}{13}\begin{bmatrix} 5 & 1 \\ -3 & 2 \end{bmatrix}$

(d) Impossible

(e) Does not exist

(f) $\begin{bmatrix} 8 & -4 & 5 \\ 8 & 9 & -7 \end{bmatrix}$

(g) $\begin{bmatrix} 1 & -7 \\ 21 & 22 \end{bmatrix}$

18. $A^{-1} = \begin{bmatrix} 0 & -2 & 5 \\ -2 & 1 & -8 \\ 1 & 0 & 3 \end{bmatrix}$

19. Using the $[A|I]$ method, show that A can not be reduced to I, a row of zeros appears in the A part during the row reduction.

20. (a) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 17 \end{bmatrix}$

    (b) $X = A^{-1}B = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

21. $(0, 1, 2)$

22. Using row reduction, the last row reduces to $[0\ 0\ 0 \mid 6]$ implying that there is no solution.

23. (a) $x = -1 - 3z$, $y = 4 - 2z$ and z is free

    (b) Infinitely many solutions

24. (a) must

    (b) can not

    (c) might

    (d) might

25. Let P_n represent the given statement. Then $P_1 : 3 = \frac{3(3^1-1)}{2}$ is true.

    Assume that P_k is true, then

    $P_k : 3 + 9 + 27 + \cdots + 3^k = \frac{3(3^k-1)}{2}$

    adding 3^{k+1} to both sides and simplifying the right hand side gives

    $3 + 9 + 27 + \cdots + 3^k + 3^{k+1} = \frac{3(3^{k+1}-1)}{2}$

    implying that P_{k+1} is true. Therefore, P_n is true for all positive integer n.