(Marks)

1. Evaluate each of the following integrals without the use of integration tables.

(3) (a) $\displaystyle\int \left(\frac{3}{\csc x} + \frac{1}{2^{-x}} + \sqrt{2x} - \pi^2 \right) dx$

(3) (b) $\displaystyle\int \frac{x^3 + x^2 - 2x + 10}{x + 3}\, dx$

(3) (c) $\displaystyle\int \ln(3x - 2)\, dx$

(4) (d) $\displaystyle\int_1^e \frac{3^{\ln x}}{x}\, dx$

(4) (e) $\displaystyle\int (x^2 - 1)\, e^{2x}\, dx$

(4) (f) $\displaystyle\int \frac{6x^2 - 7x - 2}{(x-1)^2(x+2)}\, dx$

(4) (g) $\displaystyle\int \frac{x}{\sqrt{x-3}}\, dx$

(4) 2. Use the Simpson's rule with $n = 6$ to approximate $\displaystyle\int_2^8 \frac{dx}{x^2 \ln x}$. Round your answer to four decimal places.

3. Use the table of integrals to solve each of the following. In each case, state the formula number and justify its use.

(4) (a) $\displaystyle\int \frac{3x^2}{(x^3+1)^2 - 4}\, dx$

(4) (b) $\displaystyle\int \frac{(x-1)^2}{\sqrt{x^2 - 2x + 17}}\, dx$

(8) 4. Solve the following differential equations for y:

(a) $y\, y' = e^{x - y^2}$ with condition $y(0) = 0$

(b) $\dfrac{y'}{2} = \dfrac{x\, e^{-y}}{\sqrt{x^2+3}}$ with condition $y(1) = 0$

(6) 5. A condominimum in St. Henri was bought for $\$64,000$ five years ago. The rate of appreciation in its value V at any time t (years) is proportional to the cube root of V. Currently, its valuation is $\$80,000$. When will it have a valuation of $\$100,000$?

(4) 6. Given the curves $f(x) = x^2$ and $g(x) = 4 - x^2$, determine

(a) the point(s) of intersection of $f(x)$ and $g(x)$,

(b) graph the functions $f(x)$ and $g(x)$.

(c) find the area bounded by the graph of f and g between $x = 0$ and $x = 2$.

(6) 7. Given the demand function $p_1(x) = (x-5)^2$ and the supply function $p_2(x) = x^2 + x + 3$,

(a) find the equilibrium point,

(b) sketch and identify the regions representing the consumer and producer surpluses,

(c) evaluate the producer surplus.

(6) 8. Use l'Hôpital's rule to evaluate the following limits

(Marks)

(a) $\lim_{x\to 0} \dfrac{(x+1)^2-1}{x-\sin x}$

(b) $\lim_{x\to \infty} \dfrac{x^2+1}{x\,\ln x}$

(8) 9. Evaluate each improper integral and state whether it converges or diverges

(a) $\displaystyle\int_0^4 \frac{1}{\sqrt{4-x}}\,dx$

(b) $\displaystyle\int_{-\infty}^2 \frac{2x}{(x^2+4)^{5/3}}\,dx$

(3) 10. Consider the sequence $\left\{-1\,,\ 3\,,\ \dfrac{-9}{2}\,,\ \dfrac{27}{6}\,,\ \dfrac{-81}{24}\,,\ \dfrac{243}{120}\,,\ ...\right\}$

(a) Give the next term of the sequence.

(b) Give an expression for the n^{th} term of the sequence.

(6) 11. Determine the convergence or divergence of each sequence $\{a_n\}$. If the sequence converges, find the limit.

(a) $a_n = \dfrac{n!}{(n-2)!}$

(b) $a_n = \dfrac{1+(-1)^n}{3^n}$

(6) 12. Determine with *justification* if each of the following series is convergent or divergent.
If the series is convergent, *find its sum*.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2+(-2)^n}{3^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2}{\sqrt{2n^4+3}}$

(3) 13. Given a repeated decimal $2.0\overline{4}$, express it using a geometric series, find the sum of the geometric series and write the decimal as the ratio of two integers.

(3) 14. A deposit of \$50 is made at the beginning of each week for 2 years into an account that pays an annual rate of 13% compounded weekly. Find the total amount in this account at the end of 2 years.

(4) 15. A ball is dropped from a height of 10 meters. The height of each bounce is $\frac{2}{3}$ that of the preceding bounce. Find the total vertical distance traveled by the ball.

(Marks)

ANSWERS

(1 a) $-3\cos x + \dfrac{2^x}{\ln(2)} + \frac{2\sqrt{2}}{3}\,x^{3/2} - \pi^2\,x + C$; (1 b) $\frac{1}{3}\,x^3 - x^2 + 4x - 2\ln|x+3| + C$

(1 c) $x\,\ln(3x-2) - \frac{2}{3}\,\ln(3x-2) - x + C$; (1 d) $\dfrac{2}{\ln(3)} \approx 1.82$; (1 e) $\frac{1}{2}\,(x^2-1)e^{2x} - \frac{1}{2}\,x\,e^{2x} + \frac{1}{4}\,e^{2x} + C$

(1 f) $4\,\ln|x+2| + 2\,\ln|x-1| + \dfrac{1}{x-1} + C$; (1 g) $\frac{2}{3}(x-3)^{3/2} + 6(x-3)^{1/2} + C$; (2) 0.3451

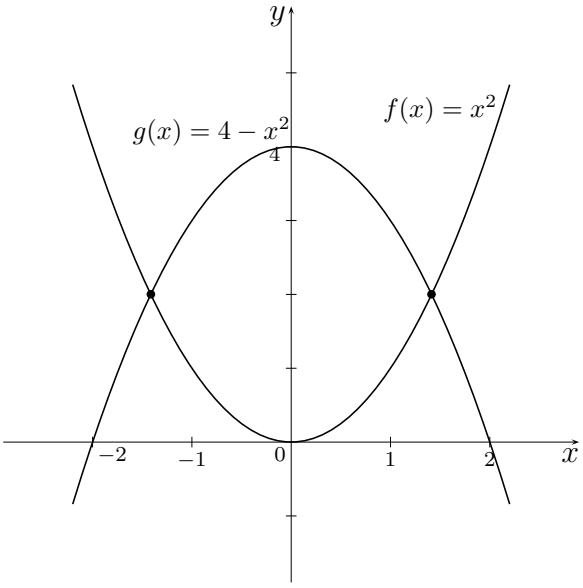
(3 a) substitution then F12: $\frac{1}{4}\,\ln\left|\dfrac{x^3-1}{x^3+3}\right| + C$

(3 b) complete the square then F28: $\frac{1}{2}\,(x-1)\sqrt{x^2-2x+17} - 8\,\ln\left|x-1+\sqrt{x^2-2x+17}\,\right| + C$

(4 a) $y = \pm\sqrt{\ln\left(2e^x - 1\right)}$; (4 b) $y = \ln\left(2\sqrt{x^2+3} - 3\right)$; (5) $t \approx 10.8$ years

(6 a) points of intersection:
$\left(-\sqrt{2}, 2\right)$; $\left(\sqrt{2}, 2\right)$

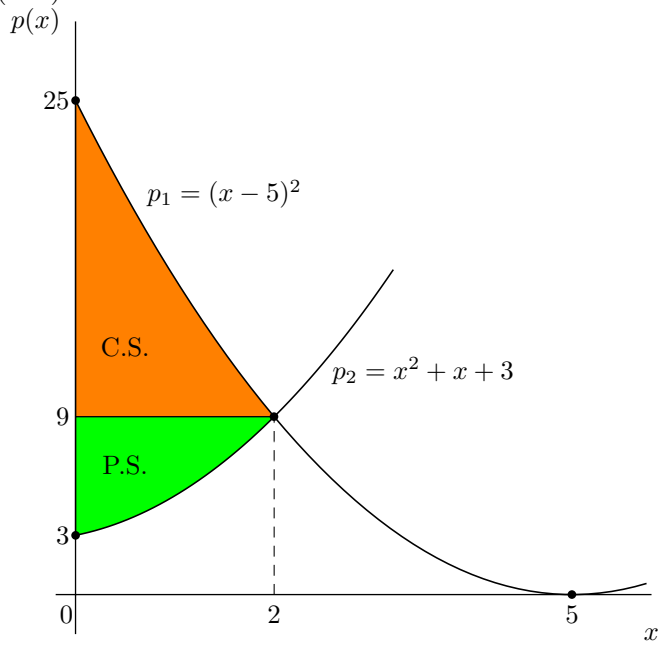
(6 b)



(6 c) Area $= \frac{8}{3} \approx 2.67$ square units

(7 a) point of equilibrium: $(2, 9)$

(7 b)



(7 c) P.S.=7.33

(8 a) ∞ ; (8 b) ∞ ; (9 a) converges to 4 ; (9 b) converges to $-\frac{3}{8} = -0.375$

(10 a) $a_7 = -\dfrac{729}{720} = -\dfrac{81}{80}$; (10 b) $a_n = (-1)^n\,\dfrac{3^{n-1}}{(n-1)!}$

(11 a) ∞ ; (11 b) 0 ; (12 a) converges to $\frac{18}{5} = 3.6$; (12 b) divergent

(13) $\dfrac{92}{45}$; (14) \$ 5945.01 ; (15) 50 meters