

Mathematical Models 1
201-115
Fall 2010
Final Examination
Instructor: Bob DeJean

Except for accuracy/ significant digit questions, please answer to 4 decimal places.

1 mark questions

Calculate to the correct accuracy or significant digits:

$$2.4 \times 10^4 + 7.41 \times 10^3 =$$

$$\frac{6.0312}{12.4} =$$

Write $\frac{3\pi}{5}$ in degrees.

Joel is floating in the Caribbean Sea, bobbing up and down some 20 cm every 35 seconds.

What is his Amplitude ?

What is his period ?

Write his equation of motion ? (keep things simple and assume it is simple harmonic motion). *(Each part is one mark)*

2 mark questions

Bill's 'Vette is 7 m long. One end is firmly on pavement, but the other is on ice, so some of his friends pushed that end, turning it by some 150° . How far did the end move ?

The buckle of Ryan's belt is a rectangle of polished steel 9 cm by 8 cm. He stands 250 cm from the campfire warming himself. What solid angle does the buckle make with the fire ?

Ravinder has designed a new sort of paper shredder. The design is quite sophisticated, depending on several lasers to reduce the paper to carbon dioxide. It looks like a garbage can 50 cm high, with a diameter of 24 cm. The top is concave, a hemisphere into which you do not put your hand if you are smart. What is the volume of Ravinder's shredder?

Solve any way you like for x and y :

$$2x - 3y = 20$$

$$5x + 2y = 31$$

Calculate: $\begin{vmatrix} 2 & 0 & 4 \\ 3 & 6 & 1 \\ 2 & 5 & -4 \end{vmatrix} =$

Set up the solution for y using Cramer's rule. Don't work it out.

$$x + 2y + 3z = 21$$

$$5x + 6y - 7z = -31$$

$$2x - 3y + 7z = 41$$

Here are two vectors.
Draw $2u + v$.



Write $\overrightarrow{(-5, 4)}$ in polar form.

Simplify: $e^{\ln x} =$

Write using simple logs: $\log\left(\frac{1666x^4}{y}\right) =$

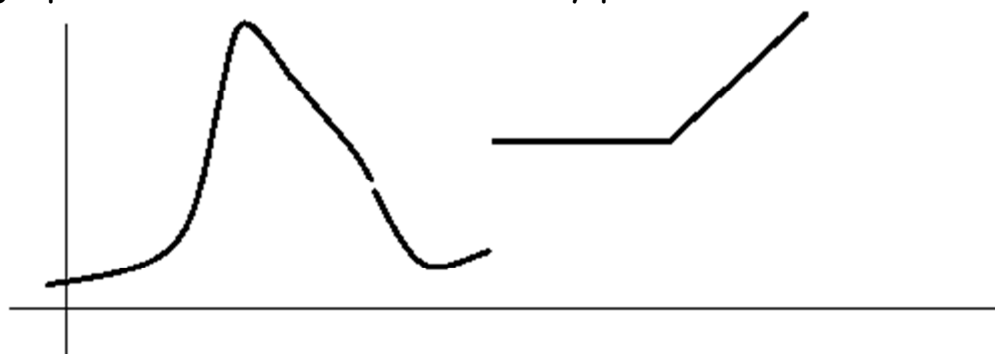
Write using one log: $\ln(x + 3) - \frac{1}{2} \ln(x - 3) =$

Solve for x in each case:

$$7(3^{x+1}) = 14\,000\,000$$

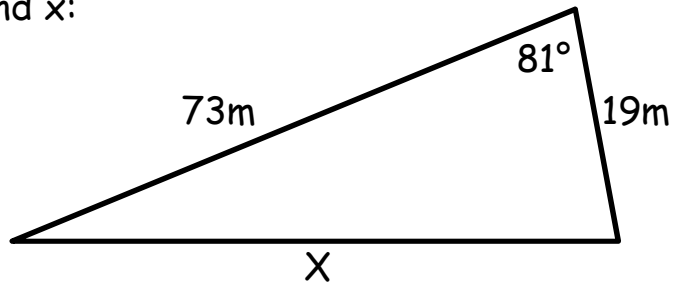
$$\log(2x + 6) - \log(x - 2) = \log 4$$

Here's the graph of a function. Are there any places where it is not continuous?

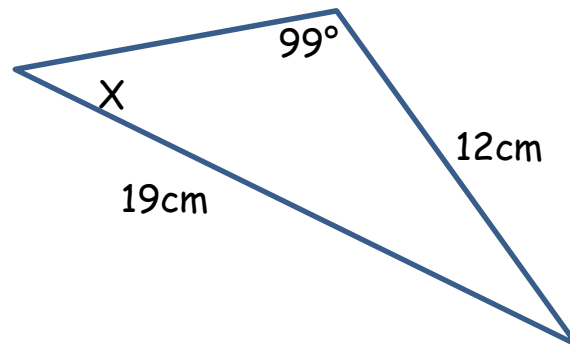


3 mark questions

Find x:



Find angle X:



Sketch the graph of $y = 12 + 4 \sin(6x + 90^\circ)$

Vertical shift is

Amplitude is

Phase Shift is

Period is

Solve for angle X: $3 \tan X - 5 = 6$ (X is between 0 and 360°)

Calculate in a simple form:

$$(3 + 2j) - 4(1 - j) =$$

$$\frac{5 + 4j}{3 - j} =$$

$$(4 \text{ cis } 20^\circ) (9 \text{ cis } 49^\circ) =$$

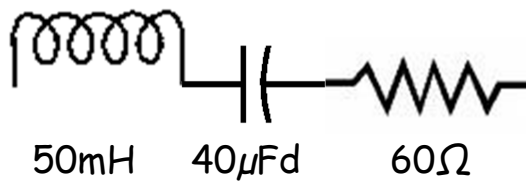
$$(2 \text{ cis } 75^\circ)^4 =$$

Find all the fourth roots of $625 \text{ cis } 200^\circ$.

Write $(6 - 8j)$ in polar form

Write $100e^{2j}$ in rectangular form.

Find the impedance across this mess, assuming a 60 Hz current.



Find:

$$\lim_{x \rightarrow \infty} \frac{3x + 5}{x - 7} =$$

$$\lim_{x \rightarrow 4} \frac{x^2 - 4x}{2x^2 - 32} =$$

Find the derivatives: Simplify if it is easy.

$$Y = 23x^3 - 333x^2 + 911$$

$$Y = 6x^2 \sin(4x) + x$$

$$y = \frac{x^3}{2x + 7}$$

$$Y = 10(2x+3)^6$$

$$y = \sqrt{\frac{1 + \sin x}{x}}$$

Find the derivatives

$$y = \tan x - \sec 4x$$

Find the derivative, that is the $\frac{dy}{dx}$, implicitly:

$$x^4 + y^4 = 3xy^3$$

Use the Limit Definition to find the derivative of $y = 7x^2$. Show your work.

Answers

31 000

0.486

108°

10 cm

35 seconds

$$y = 10 \sin(0.1795 t)$$

13.5 m

0.001 152 steradians

$$6048\pi = 19\,000 \text{ cm}^3$$

$$x = 7 \quad y = -2$$

-46

$$y = \frac{\begin{vmatrix} 1 & 21 & 3 \\ 5 & -31 & -7 \\ 2 & 41 & 7 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 3 \\ 5 & 6 & -7 \\ 2 & -3 & 7 \end{vmatrix}}$$

vector going straight right

$$6.403 / \underline{141.3^\circ}$$

x

$$\log 1666 + 4 \log x - \log y$$

$$\ln\left(\frac{x+3}{\sqrt{x-3}}\right)$$

12.2063

7

2 places: small break and big break

72.50 m

38.59°

sine wave with

VS = 12

amp = 4
 phase shift = -15°
 period = 60°
 wave "ends" at 45°

74.74° or 254.74°

$$-1 + 6j$$

$$1.1 + 1.7j$$

$$36 \text{cis} 69^\circ$$

$$16 \text{cis} 300^\circ$$

$$5 \text{cis} 50^\circ, 5 \text{cis} 140^\circ, 5 \text{cis} 230^\circ, 5 \text{cis} 320^\circ$$

$$10 \text{cis} -53.13^\circ$$

$$-41.61 + 90.93j$$

$$60 - 47.46j \Omega$$

$$3$$

$$\frac{1}{4}$$

$$69x^2 - 666x$$

$$12x \sin(4x) + 24x^2 \cos(4x) + 1$$

$$4x^3 + 21x^2$$

$$\frac{4x^3 + 21x^2}{(2x + y)^2}$$

$$120(2x + 3)^5$$

$$\left(\frac{1 + \sin x}{x}\right)^{-1/2} \left(\frac{x \cos x - 1 - \sin x}{x^2}\right)$$

$$\sec^2 x - 4 \sec(4x) \tan(4x)$$

$$\frac{dy}{dx} = \frac{3y^3 - 4x^3}{4y^3 - 9xy^2}$$

show all steps, from $\lim_{h \rightarrow 0} \frac{7(x+h)^2 - 7x^2}{h}$ to $14x$