

Mathematical Models 1  
201-115  
Fall 2010  
Final Examination  
Instructor: Bob DeJean

Except for accuracy/ significant digit questions, please answer to 4 decimal places.

*1 mark questions*

Calculate to the correct accuracy or significant digits:

$$2.4 \times 10^4 + 7.41 \times 10^3 =$$

$$\frac{6.0312}{12.4} =$$

Write  $\frac{3\pi}{5}$  in degrees.

Joel is floating in the Caribbean Sea, bobbing up and down some 20 cm every 35 seconds.

What is his Amplitude ?

What is his period ?

Write his equation of motion ? (keep things simple and assume it is simple harmonic motion). *(Each part is one mark)*

*2 mark questions*

Bill's 'Vette is 7 m long. One end is firmly on pavement, but the other is on ice, so some of his friends pushed that end, turning it by some  $150^\circ$ . How far did the end move ?

The buckle of Ryan's belt is a rectangle of polished steel 9 cm by 8 cm. He stands 250 cm from the campfire warming himself. What solid angle does the buckle make with the fire ?

Ravinder has designed a new sort of paper shredder. The design is quite sophisticated, depending on several lasers to reduce the paper to carbon dioxide. It looks like a garbage can 50 cm high, with a diameter of 24 cm. The top is concave, a hemisphere into which you do not put your hand if you are smart. What is the volume of Ravinder's shredder?

Solve any way you like for  $x$  and  $y$ :

$$2x - 3y = 20$$

$$5x + 2y = 31$$

Calculate:  $\begin{vmatrix} 2 & 0 & 4 \\ 3 & 6 & 1 \\ 2 & 5 & -4 \end{vmatrix} =$

Set up the solution for  $y$  using Cramer's rule. Don't work it out.

$$x + 2y + 3z = 21$$

$$5x + 6y - 7z = -31$$

$$2x - 3y + 7z = 41$$

Here are two vectors.

Draw  $2u + v$ .



Write  $\overrightarrow{(-5, 4)}$  in polar form.

Simplify:  $e^{\ln x} =$

Write using simple logs:  $\log\left(\frac{1666x^4}{y}\right) =$

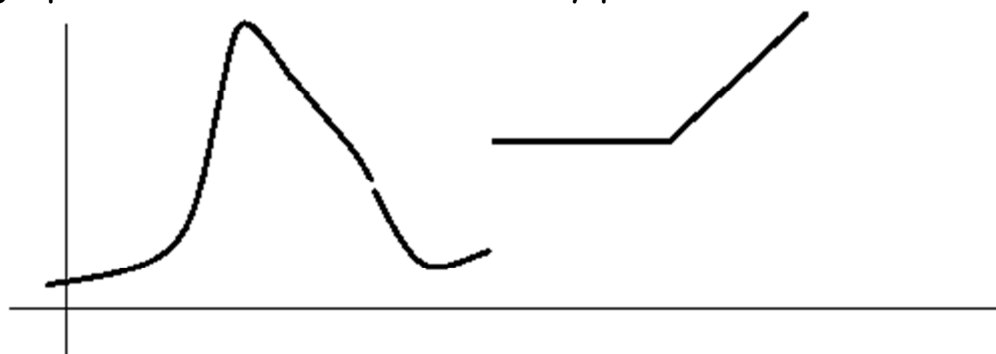
Write using one log:  $\ln(x + 3) - \frac{1}{2} \ln(x - 3) =$

Solve for  $x$  in each case:

$$7(3^{x+1}) = 14\,000\,000$$

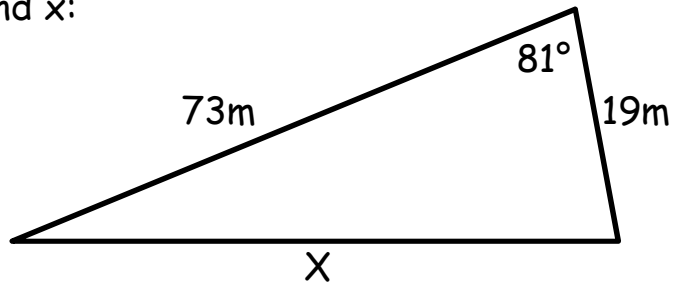
$$\log(2x + 6) - \log(x - 2) = \log 4$$

Here's the graph of a function. Are there any places where it is not continuous?

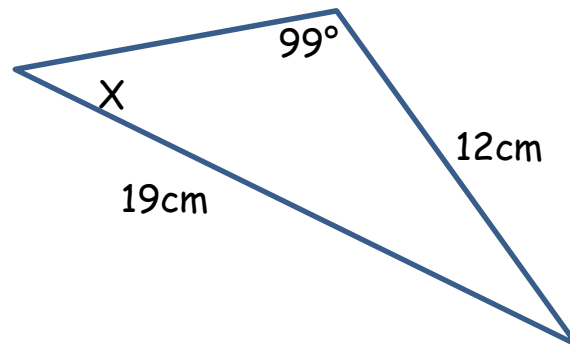


3 mark questions

Find x:



Find angle X:



Sketch the graph of  $y = 12 + 4 \sin(6x + 90^\circ)$

Vertical shift is

Amplitude is

Phase Shift is

Period is

Solve for angle X:  $3 \tan X - 5 = 6$  (X is between 0 and  $360^\circ$ )

Calculate in a simple form:

$$(3 + 2j) - 4(1 - j) =$$

$$\frac{5 + 4j}{3 - j} =$$

$$(4 \text{ cis } 20^\circ) (9 \text{ cis } 49^\circ) =$$

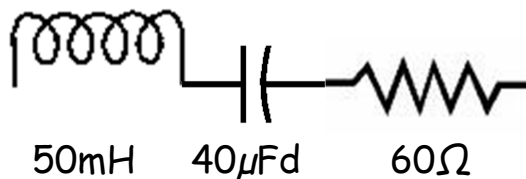
$$(2 \text{ cis } 75^\circ)^4 =$$

Find all the fourth roots of  $625 \text{ cis } 200^\circ$ .

Write  $(6 - 8j)$  in polar form

Write  $100e^{2j}$  in rectangular form.

Find the impedance across this mess, assuming a 60 Hz current.



Find:

$$\lim_{x \rightarrow \infty} \frac{3x + 5}{x - 7} =$$

$$\lim_{x \rightarrow 4} \frac{x^2 - 4x}{2x^2 - 32} =$$

Find the derivatives: Simplify if it is easy.

$$Y = 23x^3 - 333x^2 + 911$$

$$Y = 6x^2 \sin(4x) + x$$

$$y = \frac{x^3}{2x + 7}$$

$$Y = 10(2x+3)^6$$

$$y = \sqrt{\frac{1 + \sin x}{x}}$$

Find the derivatives

$$y = \tan x - \sec 4x$$

Find the derivative, that is the  $\frac{dy}{dx}$ , implicitly:

$$x^4 + y^4 = 3xy^3$$

Use the Limit Definition to find the derivative of  $y = 7x^2$ . Show your work.



Answers

31 000

0.486

$108^\circ$

10 cm

35 seconds

$$y = 10 \sin(0.1795 t)$$

13.5 m

0.001 152 steradians

$$6048\pi = 19\,000 \text{ cm}^3$$

$$x = 7 \quad y = -2$$

-46

$$y = \frac{\begin{vmatrix} 1 & 21 & 3 \\ 5 & -31 & -7 \\ 2 & 41 & 7 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 3 \\ 5 & 6 & -7 \\ 2 & -3 & 7 \end{vmatrix}}$$

vector going straight right

$$6.403 / \underline{141.3^\circ}$$

x

$$\log 1666 + 4 \log x - \log y$$

$$\ln\left(\frac{x+3}{\sqrt{x-3}}\right)$$

12.2063

7

2 places: small break and big break

72.50 m

$38.59^\circ$

sine wave with

VS = 12

amp = 4  
 phase shift =  $-15^\circ$   
 period =  $60^\circ$   
 wave "ends" at  $45^\circ$

$74.74^\circ$  or  $254.74^\circ$

$-1 + 6j$   
 $1.1 + 1.7j$   
 $36 \text{cis} 69^\circ$   
 $16 \text{cis} 300^\circ$   
 $5 \text{cis} 50^\circ, 5 \text{cis} 140^\circ, 5 \text{cis} 230^\circ, 5 \text{cis} 320^\circ$   
 $10 \text{cis} -53.13^\circ$   
 $-41.61 + 90.93j$   
 $60 - 47.46j \Omega$

3

$\frac{1}{4}$

$69x^2 - 666x$

$12x \sin(4x) + 24x^2 \cos(4x) + 1$

$4x^3 + 21x^2$

$(2x + y)^2$

$120(2x + 3)^5$

$\left(\frac{1 + \sin x}{x}\right)^{-1/2} \left(\frac{x \cos x - 1 - \sin x}{x^2}\right)$

$\sec^2 x - 4 \sec(4x) \tan(4x)$

$\frac{dy}{dx} = \frac{3y^3 - 4x^3}{4y^3 - 9xy^2}$

show all steps, from  $\lim_{h \rightarrow 0} \frac{7(x+h)^2 - 7x^2}{h}$  to  $14x$