1. Solve each of the following systems, or explain why the system has no solution.

a) $\begin{cases} x-4y-12z=17 \\ y+2z=-3 \\ 2x-5y-18z=25 \end{cases}$

b) $\begin{cases} 3x+3y+3z-21w=9 \\ -2x-y+2z+9w=-4 \\ 4x+y-7z-10w=4 \\ 2x+4y+12z-18w=11 \end{cases}$

2. Given the system $\begin{cases} x+3y+10z=9 \\ x+2y+6z=7 \\ 2x+8y+(19+k^2)z=27 \end{cases}$

Find all values of k for which the system is:

a) consistent

b) inconsistent.

3. a) Find the intersection of the planes -3x +16y +17z = -39 and x -6y -7z = 15.

b) Which of the following best describes this intersection? Circle your selection.

i) No intersection ii) Point iii) Line iv) Plane v) $\Re^3$

4. A greenhouse is used to raise plants for winter use. Among these are tomato plants, zucchini plants and cucumber plants. To insure rapid plant growth, the plants are given two types of fertilizer each week, Fertilizer A and Fertilizer B. Tomato plants require 9 units of Fertilizer A and 3 units of Fertilizer B, zucchini plants require 8 units of Fertilizer A and 4 units of Fertilizer B, cucumber plants require 6 units of Fertilizer A and 6 units of Fertilizer B. If only 252 units of Fertilizer A and 216 units of Fertilizer B are on hand, how many of each type of plant will get a proper dose of each fertilizer this week?

a) Define all the necessary variables x, y, and z, and set up the system of equations required to solve the problem. **DO NOT SOLVE.**

b) Given that the parametric solution for the problem is $(2t-60, 99-3t, t)$, find all particular solutions that are realistic.

5. Given $A=\begin{bmatrix} 6 & 2 & 2 \\ 0 & 1 & 6 \end{bmatrix}$, $B=\begin{bmatrix} 9 & -4 \\ 9 & -5 \end{bmatrix}$, $C=\begin{bmatrix} 4 & 8 \\ -1 & 9 \\ 0 & 0 \end{bmatrix}$

Find the following or explain why they don't exist:

a) $2B-AC$ b) $3A^t-C$ c) $5(B^{-1}B)^4$

6. Let A and B be 4×4 matrices such that $\det(A)=6$ and $\det(B)=2$. Find:

a) $\det[(AB)^t]$ b) $\det(BAB)$ c) $\det(3B^{-1})$ d) $\det[(BA^{-1})^2]$

7. Let a simple economy consist of two industries: Paper and Wood. The production of \$1 of paper requires 40¢ of paper and 70¢ of wood. The production of \$1 of wood requires 10¢ of paper and 20¢ of wood. There is external demand for \$8200 of paper and \$4100 of wood.

a) What dollar amount of paper and wood should be produced to meet the demand?

b) Determine whether or not each industry is profitable. Why or why not?

c) Is the economy productive? Why or why not?

8. Given $A = \begin{bmatrix} 0 & 3 & 2 \\ -2 & 7 & 1 \\ 5 & 5 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, and $B = \begin{bmatrix} 21 \\ 0 \\ -42 \end{bmatrix}$, find:

a) M_{31} b) $\det(A)$ c) $adj(A)$

d) A^{-1} using $adj(A)$ e) The solution to $AX = B$ using A^{-1}.

9. Consider the system $AX = B$, where

$$A = \begin{bmatrix} 2 & 1 & 4 & 4 \\ 9 & 1 & 1 & 2 \\ 0 & 3 & 1 & 0 \\ 7 & 2 & 0 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ 1 \\ 0 \\ -2 \end{bmatrix}$$

a) Find $\det(A)$.

b) Use Cramer's rule to find x_3 **only**.

10. Given two points $P(-3,1,4,0)$; $Q(-4,-9,2,-4)$ and the line

$L_1: x = 5 + 4t, y = 3 + 2t, z = -2 + t, w = 2 + 2t$

a) Find the vector $\overrightarrow{PQ}$.

b) Find the magnitude of $\overrightarrow{PQ}$.

c) Is the point P on the line L_1? Explain.

d) Find a vector equation of a line L_2 that passes through the point Q and is parallel to L_1.

11. Suppose A is a 4x7 matrix

a) What is the maximum value for the rank of A?

b) If dimension of Col(A) is 2, what is the dimension of Null(A)?

c) What is Dim(Col(A)) if the rank of A is 3?

d) If the reduced form of A has 3 leading ones, what is the dimension of Col(A^t)?

12. Let the set $S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \Re^3 \;\middle|\; x = -2t,\ y = 0,\ z = 3t \quad \textit{where } t \textit{ is any real number} \right\}$

a) Is $\vec{0}$ in S? Justify.

b) Is S closed under scalar multiplication? Justify.

c) Is S closed under addition? Justify.

d) Is S a subspace of $\Re^3$?

13. Let $\vec{u}_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$, $\vec{u}_2 = \begin{pmatrix} -3 \\ -6 \\ 3 \end{pmatrix}$, $\vec{u}_3 = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$, $\vec{u}_4 = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$, $\vec{u}_5 = \begin{pmatrix} 5 \\ 7 \\ -8 \end{pmatrix}$

and $U = \begin{bmatrix} 1 & -3 & -1 & 2 & 5 \\ 2 & -6 & -1 & 5 & 7 \\ -1 & 3 & 2 & 1 & -8 \end{bmatrix}$, which reduces to $R = \begin{bmatrix} 1 & -3 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

a) Find the value of k so that the vector $\vec{b} = \begin{pmatrix} k \\ 4 \\ 3 \end{pmatrix}$ is in the span of $\{\vec{u}_1, \vec{u}_3\}$.

b) Determine if the following sets of vectors are linearly dependent or independent. Briefly justify your answer.

i) $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$, ii) $\{\vec{u}_2, \vec{u}_3, \vec{u}_4\}$, iii) $\{\vec{u}_2, \vec{u}_5\}$,

c) Select a basis for Col(U) and write non basic column vectors of U as linear combinations of the basis vectors. What is the dimension of Col(U)?

d) Find a linearly independent set of vectors that spans the Null space of U. What is the nullity of U?

14. A potter is making cups and plates. It takes her 6 minutes to make a cup and 3 minutes to make a plate. Each cup uses ½ lb. of clay and each plate uses one lb. of clay. She has 20 hours available for making the cups and plates and has 250 lbs. of clay on hand. She makes a profit of $2 on each cup and $1.50 on each plate. How many cups and how many plates should she make in order to maximize her profit?

Set up the linear programming problem as follows:

a) Define all the variables (using the phrase "the number of").
b) State the objective and identify the objective function (in terms of the variables).
c) State all the constraints (in terms of the variables).
d) Solve the problem using the graphical method.

15. Use the Simplex Method to **minimize**:

$$P = -5x - 2y - 4z$$

subject to

$$\begin{aligned} 2x + 3y + z &\le 6 \\ 2x + y + z &\le 5 \\ 2x + y + 3z &\le 7 \\ x \ge 0 \quad y \ge 0 \quad z &\ge 0 \end{aligned}$$

State the minimum value of P and the corresponding basic feasible solution.

16. a) Graph the feasibility region determined by the constraints below:

$$\begin{aligned} x + y &\le 200 \\ x &\le 150 \\ y &\le 120 \\ x \ge 0 \quad y &\ge 0 \end{aligned}$$

b) Using the constraints from part a), maximize the objective function $P = 2x + 5y$ using the Simplex Method.

c) On the feasibility region from part a), trace the path used in the Simplex Method by labelling only those vertices corresponding to the basic feasible solution for each simplex table.

ANSWERS:

1. a) (5+4t, -3-2t, t) b) inconsistent
2. a) $k \neq \pm 3$ b) $k = \pm 3$
3. a) $(x, y, z) = (-3-5t, -3-2t, t)$ b) iii) Line
4. a) Let x = # of properly fertilized tomato plants
   y = # of properly fertilized zucchini plants $9x + 8y + 6z = 252$
   z = # of properly fertilized cucumber plants $3x + 4y + 6z = 216$
   b) $30 \leq t \leq 33$ therefore solutions are: (0, 9, 30), (2, 6, 31), (4, 3, 32), (6, 0, 33)
5. a) $\begin{bmatrix} -4 & -74 \\ 19 & -19 \end{bmatrix}$ b) $\begin{bmatrix} 14 & -8 \\ 7 & -6 \\ 6 & 18 \end{bmatrix}$ c) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$
6. a) 12 b) 24 c) $\frac{81}{2}$ d) $\frac{1}{9}$
7. a) \$17 000 of paper and \$20 000 of wood b) wood is profitable, paper not profitable
   c) economy is productive
8. a) -11 b) -63 c) $\begin{bmatrix} 9 & 4 & -11 \\ 9 & -10 & -4 \\ -45 & 15 & 6 \end{bmatrix}$ d) $-\frac{1}{63}\begin{bmatrix} 9 & 4 & -11 \\ 9 & -10 & -4 \\ -45 & 15 & 6 \end{bmatrix}$
   e) $(-\frac{31}{3}, -\frac{17}{3}, 19)$
9. a) -34 b) $-\frac{12}{17}$
10. a) $(-1, -10, -2, -4)$ b) 11 c) P is not on the line
    d) $(x, y, z, w) = (-4, -9, 2, -4) + t(4, 2, 1, 2)$
11. a) 4 b) 5 c) 3 d) 3
12. a) yes b) yes c) yes d) yes
13. a) $k = \frac{1}{3}$ b) i) L.D ii) L.I iii) L.I
    c) basis for Col(U) = $\{\vec{u}_1, \vec{u}_3, \vec{u}_4\}$, then $\vec{u}_2 = -3\vec{u}_1$, $\vec{u}_5 = 2\vec{u}_1 - 3\vec{u}_3$ dim(Col(U)) = 3
    d) Null(U) = $span\{(3,1,0,0,0), (-2,0,3,0,1)\}$, nullity (U) = 2
14. a) Let x = # of cups made b) Maximize $P = 2x + 1.5y$
    y = # of plates made
    c) $6x + 3y \leq 1200$
    $0.5x + y \leq 250$ $x \geq 0$, $y \geq 0$
    d) Max P = \$500 when 100 cups and 200 plates are made
15. Min P = -14 at $(2, 0, 1, 1, 0, 0)$
16. a) region with corners A(0, 0), B(0, 120), C(80, 120), D(150, 50), E(150, 0)
    b) Max P = 760 at (80, 120) c) 1- A, 2 - B, 3 - C (Max)
    (can be different; depends on steps from b))