(Marks)

(4) 1. Use the graph of the function below to find the following limits. Use ∞, $-\infty$ or DNE where appropriate.

(a) $\lim_{x\to-\infty} f(x) =$ __________

(b) $\lim_{x\to 2^-} f(x) =$ __________

(c) $\lim_{x\to 6^+} f(x) =$ __________

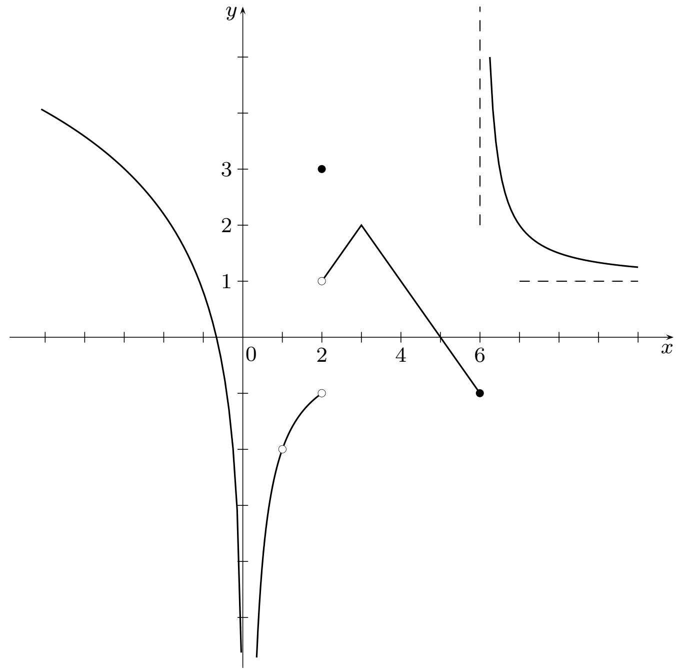
(d) $\lim_{x\to 1^-} f(x) =$ __________

(e) $\lim_{x\to 1} f(x) =$ __________

(f) $\lim_{x\to\infty} f(x) =$ __________

(g) $f(1) =$ __________

(h) $f(2) =$ __________



(15) 2. Use algebraic techniques to evaluate the following limits. Identify the limits that do not exist, and use $-\infty$ or ∞ as appropriate. Show your work.

(a) $\lim_{x\to 2} \dfrac{x^2-4}{-x^2-5x+14}$

(b) $\lim_{x\to 3^-} f(x)$, where $f(x) = \begin{cases} 7-x^2 & \text{if} \quad x<3 \\ 2x-4 & \text{if} \quad x\geq 3 \end{cases}$

(c) $\lim_{x\to+\infty} \dfrac{(2x-1)(x+2)}{x-5}$

(d) $\lim_{x\to 5^+} \dfrac{4x-2}{2x^2-7x-15}$

(e) $\lim_{x\to 3} \dfrac{\sqrt{x+1}-2}{x-3}$

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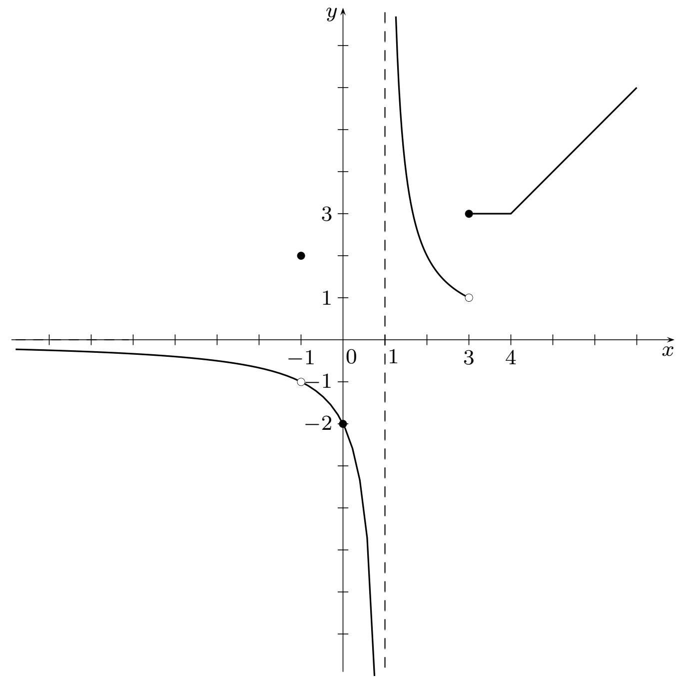
(3) 3. Given the graph of $y = f(x)$



(a) Give the interval(s) where the slope of the tangent line to the curve of $f(x)$ is negative.

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(b) Locate the x-value(s) where $f(x)$ is continuous but **not** differentiable.

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(c) Locate the x-value(s) where $f(x)$ is **not** differentiable.

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(3) 4. Find the value(s) of k such that $f(x)$ is continuous:

$$f(x) = \begin{cases} kx^2 + 2k^2x - 4 & \text{if } x \le 1 \\ 4kx^2 + k^2x + 6 & \text{if } x > 1 \end{cases}$$

(5) 5. Using only the limit definition of the derivative, show that if $f(x) = x^2 - 5x + 3$ then $f'(x) = 2x - 5$.

(28) 6. Find $\dfrac{dy}{dx}$ for each of the following functions. **Do not simplify your answers.**

(a) $y = 8x^3 - \sqrt[5]{x} + 2x^{e+1} + e^3$

(b) $y = \left(\dfrac{5x-3}{2-7x}\right)^2$

(c) $y = e^{\sec(5x)} \cot(7x)$

(d) $y = \ln\left(\dfrac{\sqrt[4]{2x-3}}{\cos(x)\,(3x^2-x)^3}\right)$

(e) $xy = (x+3y)^4 + 7$

(f) $y = 4^x \log_4(\sqrt{x})$

(g) $y = (3x+2)^{\cos(x)}$

(4) 7. Find the second derivative of $f(x) = xe^{5-6x} + x^{-1}$.

(4) 8. Find the absolute maximum and absolute minimum of $f(x) = x^3 + 2x^2 - 15x + 27$ on the interval $[-4, -2]$.

(4) 9. Find an equation of the line tangent to the curve of $y = \dfrac{x^2+1}{x-2}$ at $x = 4$.

(4) 10. Use the second derivative test to find the relative extrema of $f(x) = x^4 - 4x^3 + 4x^2 + 3$.

(Marks)

(10) 11. Given $f(x) = \dfrac{(5x+4)(x-4)}{x^2}$, $f'(x) = \dfrac{16(x+2)}{x^3}$ and $f''(x) = \dfrac{-32(x+3)}{x^4}$,

(a) Find the y-intercept, x-intercepts, vertical and horizontal asymptotes, relative extrema and points of inflection (if any). Find the intervals where f is increasing, decreasing, concave up, concave down.

(b) Sketch a graph of $f(x)$.

(5) 12. For a certain product, the demand function is $p = 1000 - x$, and the average cost is $\overline{C} = \dfrac{3000}{x} + 20$.

(a) Write the revenue function in simplified form.

(b) Write the cost function in simplified form.

(c) Write the profit function in simplified form.

(d) Find the marginal profit at $x = 300$, and interpret the result.

(e) At what level of production will the profit be maximized? (Be sure to use a test to confirm that the profit is a maximum)

(6) 13. A private school wants to enclose a rectangular courtyard of 7200 square meters using the school building as one side of the yard. The opposite side to the building costs \$90 per linear meter and the other two sides cost \$40 per linear meter. The building side needs no fence. Find the dimensions of the courtyard that will result in the minimum cost of the fence.

(Be sure to use a test to confirm that this is a minimum)

(5) 14. The demand function for a manufacturer's product is given by $p = 300 - x^2$, where p is the price per unit when x units are demanded.

(a) Determine the price elasticity of the demand when $x = 5$.

(b) For what value(s) of x does the demand have unit elasticity?

Answers

(1a) $+\infty$; (1b) -1 ; (1c) $+\infty$; (1d) -2 ; (1e) -2 ; (1f) 1 ; (1g) DNE ; (1h) 3

(2a) $-\frac{4}{9} \approx -0.44$; (2b) -2 ; (2c) $+\infty$; (2d) $+\infty$; (2e) $\frac{1}{4} = 0.25$

(3a) $]-\infty, -1[\cup]-1, 1[\cup]1, 3[$; (3b) $x = 4$; (3c) $x = -1$, $x = 1$, $x = 3$, $x = 4$; (4) $k = -2$, $k = 5$

(5) Use $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to find $f'(x) = 2x - 5$

(6a) $\dfrac{dy}{dx} = 24x^2 - \frac{1}{5}x^{-4/5} + 2(e+1)x^e$; (6b) $\dfrac{dy}{dx} = 2\left(\dfrac{5x-3}{2-7x}\right)\left[\dfrac{5(2-7x)-(5x-3)(-7)}{(2-7x)^2}\right]$

(6c) $\dfrac{dy}{dx} = e^{\sec(5x)}\left[\sec(5x)\tan(5x).(5)\right]\cot(7x) + e^{\sec(5x)}\left[-\csc^2(7x).(7)\right]$

(6d) $\dfrac{dy}{dx} = \dfrac{2}{4(2x-3)} - \dfrac{-\sin(x)}{\cos(x)} - \dfrac{3(6x-1)}{3x^2-x}$; (6e) $\dfrac{dy}{dx} = \dfrac{4(x+3y)^3 - y}{x - 12(x+3y)^3}$

(Marks)

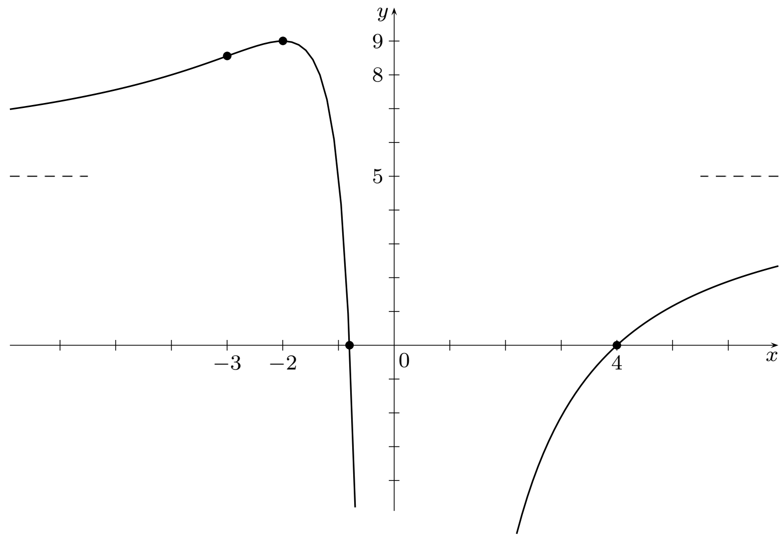
(6f) $\frac{dy}{dx} = 4^x \ln(4) \,.\, \log_4 \sqrt{x} + \frac{1}{2} \frac{1}{x\ln(4)} \,.\, 4^x$; (6g) $\frac{dy}{dx} = (3x+2)^{\cos x} \left[-\sin(x) \ln(3x+2) + \frac{3\cos(x)}{3x+2} \right]$

(7) $f''(x) = -12e^{5-6x} + 36x\, e^{5-6x} + 2x^{-3}$
(8) absolute maximum is 63 when $x = -3$; absolute minimum is 55 when $x = -4$
(9) $y = -\frac{1}{4}x + \frac{19}{2}$; (10) relative minimum at $(0,3)$ and $(2,3)$; relative maximum at $(1,4)$

(11a)

$x-$ intercepts : $\left(-\frac{4}{5}, 0\right)$; $(4,0)$
$y-$ intercept: none
vertical asymptote: $x = 0$
horizontal asymptote: $y = 5$
relative minimum: $(-2, 9)$
points of inflection: $\left(-3, \frac{77}{9}\right)$
increasing: $x < -2$ or $x > 0$
decreasing: $-2 < x < 0$
concave up: $x < -3$
concave down: $-3 < x < 0$ or $x > 0$

(11b)



(12a) $R(x) = 1000x - x^2$; (12b) $C(x) = 3000 + 20x$; (12c) $P(x) = -x^2 + 980x - 3000$
(12d) an increase from 300 to 301 units would result in an increase of about 380 \$ in profits
(12e) 490 units to maximize profit

(13) A courtyard with dimensions 90 meters by 80 meters to minimize cost

(14a) $\eta(5) = -5.5$; (14b) demand has unit elasticity at $x = 10$ units