(5) 1. Write the solution to the system $\begin{cases} 2x_1 - 4x_2 - 2x_3 + 8x_4 = -4 \\ -3x_1 + 4x_2 - x_3 - 2x_4 = 0 \\ -x_1 + 3x_2 + 3x_3 - 9x_4 = 5 \end{cases}$

(5) 2. Given the following matrix $A = \begin{bmatrix} 3 & 5 & 0 & 4 \\ -1 & 3 & 1 & -2 \\ 0 & k & 0 & 1 \\ 0 & 4 & 0 & k \end{bmatrix}$

(a) Find $|A|$ in terms of k.

(b) For what values of k is A non-invertible?

(4) 3. Find the inverse of $\begin{bmatrix} 2 & -3 & -14 \\ 1 & -2 & -7 \\ -3 & 5 & 22 \end{bmatrix}$

(5) 4. Given the following matrix $A = \begin{bmatrix} 3 & 2 & 6 \\ 9 & 4 & 22 \\ -12 & -12 & -11 \end{bmatrix}$

(a) Write A as the product of a lower triangular matrix L and an upper triangular matrix U.

(b) What is $|A|$?

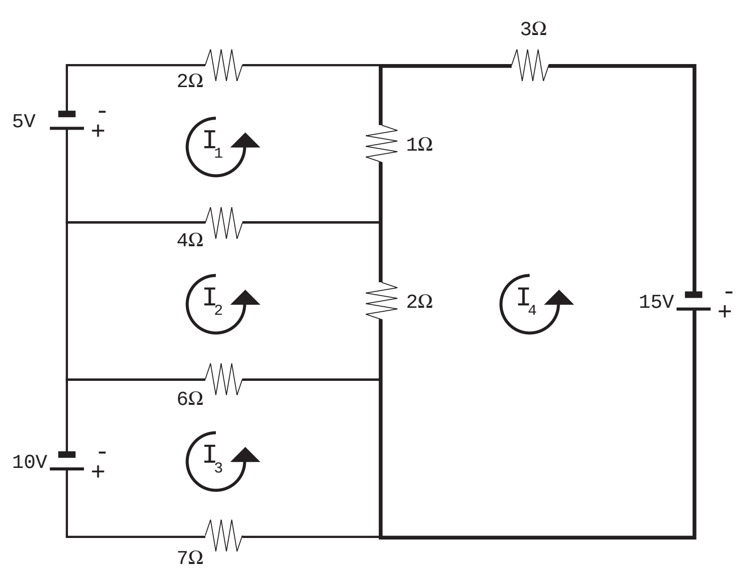
(6) 5. Consider the following block matrix:

$$M = \begin{bmatrix} 0 & B & 0 \\ 0 & 0 & A \\ I & 0 & 0 \end{bmatrix}$$

(a) Given that A and B are invertible, find the block matrix form for M^{-1}

(b) Use part (a) to find the inverse of $\begin{bmatrix} 0 & 0 & 8 & 13 & 0 & 0 & 0 \\ 0 & 0 & 3 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(4) 6. Set up an augmented matrix for finding the loop currents of the following electrical network.
You do not have to solve the system.



(5) 7. Let A be a 4×4 matrix with $|A| = -3$. Let B be a 4×4 non invertible matrix. For each part, either provide an answer or write "not enough information".

(a) What the value of $|2A|$?

(b) What is the value of $|AB|$?

(c) What is the value of $|A + B + I|$?

(d) What is the value of $|(A^T A)^{-1}|$?

(6) 8. Let A and B be $n \times n$ matrices and suppose AB is its own inverse. (That is, $(AB)^{-1} = AB$.)

(a) The inverse of BAB is: (choose the correct answer)

(1) ABA (3) AB (5) A

(2) BAB (4) BA (6) B

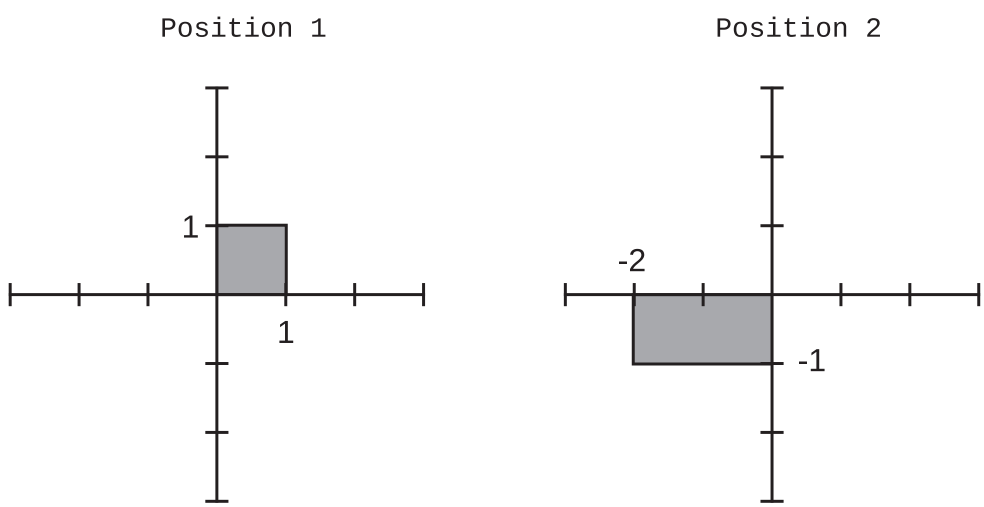
(b) Is matrix B necessarily invertible? Justify your answer.

(c) Prove that BA is also its own inverse.

(d) Evaluate and simplify $(AB + I)(AB + I)$.

(e) What is $(AB + I)^8$?

(4) 9. Find both 2×2 matrices A such that the transformation $T(\mathbf{x}) = A\mathbf{x}$ transforms the unit square from position 1 to the rectangle in position 2 (as seen below).



(7) 10. Given that

$$A = \begin{bmatrix} 2 & 4 & 20 & 7 & 0 & 20 & 17 \\ 2 & -4 & -4 & -11 & -12 & -12 & -21 \\ 1 & 0 & 4 & -1 & -3 & 2 & -1 \\ -2 & 3 & 1 & 6 & 5 & -3 & 8 \end{bmatrix} \quad \text{row reduces to}$$

$$R = \begin{bmatrix} 1 & 0 & 4 & 0 & -1 & 6 & 2 \\ 0 & 1 & 3 & 0 & -3 & -5 & -2 \\ 0 & 0 & 0 & 1 & 2 & 4 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for the column space of A

(b) Find a basis for the row space of A

(c) Find a basis for the null space of A

(d) What is rank(A)?

(e) What is dim(Nul(A))?

(f) What is rank(A^T)?

(g) What is $\dim(\mathrm{Nul}(A^T))$?

(5) 11. Let H be the set of all 2×2 matrices such that the sum of all entries is zero.

(a) Provide an example of an invertible matrix in H.

(b) Find a basis for this subspace of $M_{2\times 2}$

(c) What is the dimension of H?

(8) 12. Let $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : |x| = |y| \right\}$ be a subset of $\mathbf{R}^2$.

(a) Does H satisfy closure under vector addition? Justify.

(b) Does H contain the zero vector of $\mathbf{R}^2$? Justify.

(c) Does H satisfy closure under scalar multiplication? Justify.

(d) Is H a vector subspace of $\mathbf{R}^2$? Justify.

(8) 13. Find a specific example for each of the following:

(a) a 2×2 matrix A such that $\mathrm{Col}(A) = \mathrm{Nul}(A)$.

(b) a 3×3 matrix A with every entry different such that $|A| = 0$

(c) two orthogonal vectors in $\mathbf{R}^3$ that have no zero entries.

(d) A line in $\mathbf{R}^3$ which is parallel to the xy-plane..

(9) 14. Let $\mathcal{P}_1$ be the plane $2x + 3y + 3z = -8$
Let $\mathcal{P}_2$ be the plane $x + 2y + 2z = -6$
Let $\mathcal{P}_3$ be the plane $x + 2y + 2z = 1$

(a) Find the equation of the line of intersection between $\mathcal{P}_1$ and $\mathcal{P}_2$.

(b) What is the cosine of the angle between $\mathcal{P}_1$ and $\mathcal{P}_2$?

(c) Find the distance from $\mathcal{P}_2$ to $\mathcal{P}_3$.

(10) 15. Let $\mathcal{P}$ be the plane containing the points $Q(1, 2, 3), R(2, 3, 3)$ and $S(6, 4, -2)$.

(a) Find a normal vector to $\mathcal{P}$.

(b) Find an equation for the plane $\mathcal{P}$ (in standard form $ax + by + cz = d$).

(c) Find the area of triangle QRS.

(d) Find the volume of the parallelepiped defined by the edges OQ, OR, OS, where O is the origin.

(4) 16. Find the point of intersection between the plane $3x - 2y + 5z = 3$ and the line $\mathbf{x} = \begin{bmatrix} -2 \\ -4 \\ 8 \end{bmatrix} + t \begin{bmatrix} 2 \\ 2 \\ -3 \end{bmatrix}$

(5) 17. Let $T : V \to W$ be a one-to-one linear transformation.

(a) Write the definition of "linearly independent". Be precise.

(b) Let $\{v_1, v_2, \ldots, v_k\}$ be a linearly independent set in V above.
Prove that $\{T(v_1), T(v_2), \ldots, T(v_k)\}$ is also linearly independent.