(3) 1. What is the equation of the tangent line to the graph of $y = x^2 \arcsin(x)$ at the point where $x = 1/2$?

(30) 2. Evaluate the integrals.

(a) $\displaystyle\int x^5 \sqrt{x^3 - 1}\, dx$ (b) $\displaystyle\int \tan^3(t) \sec^3(t)\, dt$ (c) $\displaystyle\int_0^{1/3} \arctan(3x)\, dx$

(d) $\displaystyle\int \frac{x^3 + x - 2}{x^3 + x}\, dx$ (e) $\displaystyle\int \frac{x + 5}{x^2 + 6x + 13}\, dx$ (f) $\displaystyle\int \frac{\sqrt{4 - x^2}}{x^2}\, dx$

(10) 3. Evaluate the improper integrals.

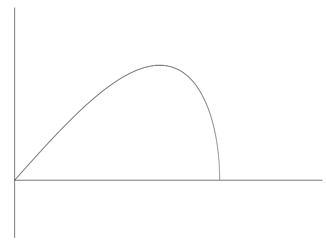
(a) $\displaystyle\int_2^{\infty} \frac{\operatorname{arcsec} x}{x\sqrt{x^2 - 1}}\, dx$ (b) $\displaystyle\int_{\mathrm{e}}^{\mathrm{e}^2} \frac{\ln x\, dx}{\sqrt{x \ln x - x}}$

(9) 4. Evaluate the limits.

(a) $\displaystyle\lim_{x \to 0} (1 + \sin 2x)^{\cot x}$ (b) $\displaystyle\lim_{x \to \pi/2} \frac{\ln(\sin x)}{(\pi - 2x)^2}$ (c) $\displaystyle\lim_{x \to 0^+} \csc(x) \arctan(x)$

(4) 5. Find the area between $y = 2x^2$ and $y = x^2(x - 1)$.

(7) 6. Let $\mathcal{R}$ be the region in quadrant I (so $x \geq 0$) between the x-axis and $y = \sqrt{4x^2 - x^4}$.



(a) Set up the integrals required to compute the volume of the solid obtained by rotating $\mathcal{R}$ about (i) the y-axis, and (ii) the line $y = -3$.

(b) Compute the volume of the solid obtained by rotating $\mathcal{R}$ about the x-axis.

(4) 7. Solve the differential equation: $\mathrm{e}^{-x} y\, y' = x$; $y(0) = -1$. Express y as a function of x.

(3) 8. Does the sequence $\{n - \ln n\}$ converge? If so, find its limit as $n \to \infty$. Justify your answer.

(9) 9. Determine whether each of the following series converges or diverges. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$ (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{\sqrt{n}} - \frac{1}{n^2} \right)$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2\, 3^n}{(2n)!}$

(7) 10. Label each series as absolutely convergent, conditionally convergent, or divergent. Justify your answers.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(\sqrt[3]{n})}$ (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin(1/n)}{\sqrt{n}}$

(3) 11. Mark each statement as TRUE (if it is necessarily true) or FALSE (otherwise). Justify your answers.

Supposing $\displaystyle\lim_{n \to \infty} a_n = 1$:

(a) then $\displaystyle\sum_{n=1}^{\infty} (1 - a_n)^n$ converges. (b) then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

(c) then $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges.

(4) 12. Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n+1}\, (x - 1)^n}{(n+1)!}$.

(5) 13. Find the Taylor series of $f(x) = x \ln x$ centered at $x = 1$. For what values of x does the Taylor series converge?

(2) 14. Let $p(x)$ be a polynomial of degree ≤ 3 so that $p(0) = p(3) = 3$.

Suppose that the antiderivative $\displaystyle\int \frac{p(x)}{x^2(x - 3)^2}\, dx$ has no terms involving the ln (logarithm) function.

(a) Determine the partial fraction decomposition of $\dfrac{p(x)}{x^2(x - 3)^2}$.

(b) Calculate $\displaystyle\int \frac{p(x)}{x^2(x - 3)^2}\, dx$.

Answers

1. $y = \left(\frac{\pi}{6} + \frac{1}{2\sqrt{3}}\right)\left(x - \frac{1}{2}\right) + \frac{\pi}{24}$
2. The integrals:
   (a) $\frac{2}{3}\left(\frac{1}{5}(x^3-1)^{5/2} + \frac{1}{3}(x^3-1)^{3/2}\right) + C$
   (b) $\frac{1}{5}\sec^5 t - \frac{1}{3}\sec^3 t + C$
   (c) $\frac{\pi}{12} - \frac{1}{6}\ln 2$
   (d) $x - 2\ln|x| + \ln(x^2+1) + C$
   (e) $\frac{1}{2}\ln(x^2+6x+13) + \arctan(\frac{x+3}{2}) + C$
   (f) $-\frac{\sqrt{4-x^2}}{x} - \arcsin(x/2) + C$
3. The improper integrals:
   (a) $\frac{5}{72}\pi^2$ (b) $2\,\mathrm{e}$
4. The limits:
   (a) e^2 (b) $-\frac{1}{8}$ (c) 1
5. 27/4
6. The volumes:
   (a) (i) $2\pi\int_0^2 x\sqrt{4x^2-x^4}\,dx$ (ii) $\pi\int_0^2 \left((3+\sqrt{4x^2-x^4})^2 - 3^2\right)\,dx$
   (b) $\frac{64}{15}\pi$
7. $y = -\sqrt{2\,\mathrm{e}^x(x-1)+3}$
8. Diverges ($n(1-\frac{\ln n}{n}) \to \infty$)
9. The series:
   (a) Converges (to 1 by TS, or LCT) (b) Diverges (LCT with $\sum \frac{1}{\sqrt{n}}$) (c) Converges (RT)
10. Alternating series:
    (a) CC (log pS so not AC; AST) (b) AC (LCT with $\sum \frac{1}{n\sqrt{n}}$)
11. T/F questions:
    (a) T ($\sqrt[n]{}$T) (b) F (nTT) (c) F (LCT with $\sum \frac{1}{n}$)
12. Power series: Radius ∞, IofC $(-\infty, \infty)$
13. Taylor series: $(x-1) + \sum_{n=2}^{\infty} \frac{(-1)^n(x-1)^n}{n(n-1)}$ IofC: $[0,2]$
14. Partial fractions:
    (a) $\frac{p(x)}{x^2(x-3)^2} = \frac{1}{3}\frac{1}{x^2} + \frac{1}{3}\frac{1}{(x-3)^2}$ (b) $-\frac{1}{3}\left(\frac{1}{x} + \frac{1}{x-3}\right) + C$