(Marks)

(6) 1. For the function $f(x) = x\ln(x)$:

(a) find the 4^{th} degree Taylor polynomial $T_4(x)$ around $x = 1$.

(b) Use Taylor's Inequality (or Lagrange's Remainder) to estimate the error in using $T_4(x)$ to approximate $f(x)$ on the interval $[0.5, 1.5]$.

(4) 2. Find the Maclaurin series for the function $f(x) = \dfrac{1}{\sqrt{1+x^3}}$. What is its radius of convergence?

(8) 3. Let $g(x) = \displaystyle\int_0^x \frac{t^2\,dt}{1+t^4}$

(a) Find the Maclaurin series for $g(x)$, and its radius of convergence;

(b) approximate $\displaystyle\int_0^{1/2} \frac{t^2\,dt}{1+t^4}$ within an error of $\pm 10^{-4}$ (and justify your answer).

(c) Find $g^{(7)}(0)$.

(6) 4. Sketch (on the same axes) the graphs of $r = 3\sin\theta$ and $r = 1 + \sin\theta$.

(a) Find all points of intersection.

(b) Set up (*but do not evaluate*) the integrals needed to find

i. the area of the region common to both (*i.e.* inside) $r = 3\sin\theta$ and $r = 1 + \sin\theta$, and

ii. the perimeter (length) of $r = 1 + \sin\theta$.

(7) 5. Suppose that a plane curve $\mathcal{C}$ given by parametric equations in t passes through the point $(0, 2)$ at $t = 1$, and satisfies $\frac{dx}{dt} = \frac{2}{t}$ and $\frac{dy}{dt} = 1 - \frac{1}{t^2}$.

(a) Find the parametric equations for $\mathcal{C}$ (*i.e.* for x and y).

(b) Find the Cartesian equation for $\mathcal{C}$ by eliminating the parameter t.

(c) Find the length of $\mathcal{C}$ from $t = 1$ to $t = 3$.

(10) 6. A space curve $\mathcal{C}$ is defined by the vector equation $\boldsymbol{r}(t) = \langle t^3, 3t^2, 6t\rangle$.

(a) Compute the velocity $\boldsymbol{v}$, acceleration $\boldsymbol{a}$, and speed v of a point moving along $\mathcal{C}$.

(b) Find the tangential and normal components of acceleration a_T, a_N, the unit tangent vector $\boldsymbol{T}$ and the unit normal vector $\boldsymbol{N}$.

Simplify your answers.

(9) 7. Identify and sketch the following. Show all your work.

(a) The surface $z^2 = r^2$.

(b) The surface $\rho = 4\cos\varphi$.

(c) The graph of the function $z = \sqrt{4 - x^2 + y^2}$.

(9) 8. (a) Calculate $f_{xy}(x, y)$ for the function $f(x, y) = x\,e^{x^2 - y^2}$.

(b) Let $z = f(x - y, y - x)$. Show that $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 0$

(c) Given $z = f(x, y)$ is implicitly defined by the equation $z = e^x \sin(y + z)$, find $\frac{\partial z}{\partial x}$.

(Marks)

(8) 9. Let $\mathcal{S}$ be the level surface $f(x,y,z) = x - y^3 - 2z^2 = 2$, and $P_0(-4,-2,1)$ a point on $\mathcal{S}$. Find:

(a) the equation of the tangent plane to $\mathcal{S}$ at P_0;

(b) the derivative of f at P_0 in the direction $\boldsymbol{v} = \langle 3, 6, -2\rangle$;

(c) the direction and value of the maximal rate of increase of f at P_0;

(d) the parametric equations of the tangent line at P_0 to the curve of intersection of $\mathcal{S}$ and the plane $2x - 3y - z = -3$.

(4) 10. Given $z = f(x,y) = \ln(2y - x)$:

(a) find the total differential dz;

(b) use dz to find an approximate value of $f(3.1, 1.98)$.

(5) 11. Find and classify all local extrema of $f(x,y) = 4xy - x^4 - y^4$.

(6) 12. Use Lagrange Multipliers to determine the dimensions of a rectangular box with no top, having a volume of 32 cubic meters and requiring the least amount of material for construction.

(12) 13. Evaluate (change the coordinates or the order of integration as appropriate):

(a) $\displaystyle\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx\, dy$ (b) $\displaystyle\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy\, dx$

(c) $\displaystyle\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx.$

(6) 14. Sketch the solid region $\mathcal{S}$ above the xy plane and inside both the hemisphere $z = \sqrt{25 - x^2 - y^2}$ and the cylinder $x^2 + y^2 = 9$.
Set up *(but do not evaluate)* triple integrals representing the volume of $\mathcal{S}$ in

(a) cartesian coordinates (b) cylindrical coordinates (c) spherical coordinates