1. (a) $T_4(x) = (x-1) + \frac{1}{2}(x-1)^2 - \frac{1}{6}(x-1)^3 + \frac{1}{12}(x-1)^4$

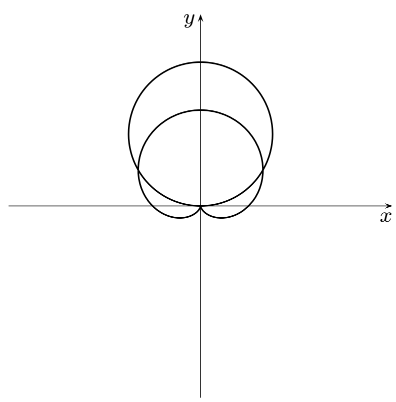
   (b) $R_4(x) = \frac{-6(x-1)^5}{z^4 5!}$ where z is between 1 and x.

   $|R_4(x)| \leq \frac{1}{40} = 0.025$ since $0.5 < z < 1.5$

2. $(1+x^3)^{-1/2} = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n (1)(3)(5)\cdots(2n-1)}{2^n n!} x^{3n}$ $\qquad R = 1$

3. (a) $g(x) = \int_0^x t^2 \sum_{n=0}^{\infty} (-t^4)^n \, dt$ which simplifies to
   $g(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+3}}{4n+3} \, dt = \frac{x^3}{3} - \frac{x^7}{7} + \frac{x^{11}}{11} - \cdots$ with $R = 1$
   (b) $g(1/2) = \frac{1}{3(2^3)} - \frac{1}{7(2^7)} \simeq 0.04055$ with error$= \pm 0.44 \times 10^{-4}$
   (c) $g^{(7)}(0) = -6! = -720$

4. (a) The points of intersection are $(3/2, \pi/6)$, $(3/2, 5\pi/6)$, and the pole.



   (b) (i) $\mathcal{A} = \int_0^{\pi/6} 9\sin^2\theta d\theta + \int_{\pi/6}^{\pi/2} (1+\sin\theta)^2 d\theta$

   (ii) $\mathcal{L} = \int_0^{2\pi} \sqrt{2(1+\sin\theta)} d\theta$ or equivalent

5. (a) $x = 2\ln t, \quad y = t + \frac{1}{t}$ where $t > 0$.

   (b) $y = e^{x/2} + e^{-x/2}$

   (c) $\mathcal{L} = \frac{8}{3}$ unit

6. $\mathbf{v}(t) = \langle 3t^2, 6t, 6\rangle \qquad \mathbf{a}(t) = \langle 6t, 6, 0\rangle \qquad$ and $v = 3(t^2+2)$.

   $a_T = 6t$ and $a_N = 6$

   $$\mathbf{T} = \left\langle \frac{t^2}{t^2+2}, \frac{2t}{t^2+2}, \frac{2}{t^2+2} \right\rangle$$

   $$\mathbf{N} = \left\langle \frac{2t}{t^2+2}, \frac{-t^2+2}{t^2+2}, \frac{-2t}{t^2+2} \right\rangle$$

7. (a) Circular cone; (b) Sphere of center $(0,0,2)$ and radius 2; (c) Hyperboloid of one sheet, $z \geq 0$ part, with y-axis as its axis

8. (a) $f_{xy} = -2y(1+2x^2)e^{x^2-y^2}$

   (b) show that $\partial z/\partial x = \partial z/\partial u - \partial z/\partial v = -\partial z/\partial y$

   (c) $\frac{\partial z}{\partial x} = \frac{e^x \sin(y+z)}{1 - e^x\cos(y+z)}$

9. (a) $x - 12y - 4z = 16$, $\quad$ (b) $D_{\mathbf{v}} f(P_0) = -61/7$, $\quad$ (c) the direction is $\frac{1}{\sqrt{161}}\langle 1, -12, -4\rangle$ and maximum rate is $\sqrt{161}$

(d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$

10. (a) $dz = \frac{1}{x-2y}dx + \frac{2}{2y-x}dy,$ (b) $f(3.1, 1.98) \simeq -0.14$

11. $(0,0)$ is a saddle point and $(-1,-1)$ and $(1,1)$ are local maxima

12. $(4,4,2)$

13. (a) $I = \frac{e^{16}-1}{4}$ (b) $I = \frac{16}{9}$ (c) $I = 8\pi$

14. (a) $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{\sqrt{25-x^2-y^2}} dzdydx$

(b) $\int_{0}^{2\pi} \int_{0}^{3} \int_{0}^{\sqrt{25-r^2}} rdzdrd\theta$

(c) $\int_{0}^{2\pi} \int_{0}^{\arctan(3/4)} \int_{0}^{5} \rho^2 \sin\phi d\rho d\phi d\theta + \int_{0}^{2\pi} \int_{\arctan(3/4)}^{\pi/2} \int_{0}^{3\csc\phi} \rho^2 \sin\phi d\rho d\phi d\theta$