Mathematical Models II
201-225-AB
Final Examination
Winter 2010

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Please answer any questions with decimal answers to 4 decimal places.

2 mark questions

The current through an 20 H inductor is given by I = 25 sin(4πt). What is the Root-Mean-Square of this current ?

Does $y = \frac{x}{x+1}$ satisfy the equation $x(x+1)y' = y$?

3 mark questions

Find the derivatives of the following functions:

$y = \frac{\sin 3x}{x}$

$y = \tan(\ln(3x))$

Find the derivatives

$y = \sec\sqrt{x}$

$y = \tan^{-1}(x-5)$

$y = x^3 \ln(x-1)$

$y = \left(e^x - 2\right)^5$

4 mark questions

Differentiate implicitly: sin x – ln y = xy

Find the equation of the line that is normal to y = $\sin^{-1}$ x at the point (0.6, 0.6435)

Use Newton's Method to solve $x^5 + 6x - 100 = 0$. Your answer should be accurate to 4 decimal places.

Daniel has a pet amphibian, a skink actually. He had a lot of trouble getting the special kind of fence that skinks cannot climb, but he was able to get about 16m of fence. He wants to build a rectangular pen for his skink. Three sides will be fenced; the fourth side will be the wall of his house, which is coated with Teflon. What dimensions will give the skink the maximum possible area to run around ?

Integrate the following:

$$\int x^{-3} + \sqrt[3]{x}\,dx =$$

$$\int_1^2 \frac{dx}{(3x-1)^4} =$$

$$\int \frac{\sec^2 x}{1+\tan x}\,dx =$$

$$\int_1^4 e^{2x-1}\,dx =$$

$$\int \sec x \tan x\, e^{\sec x}\,dx =$$

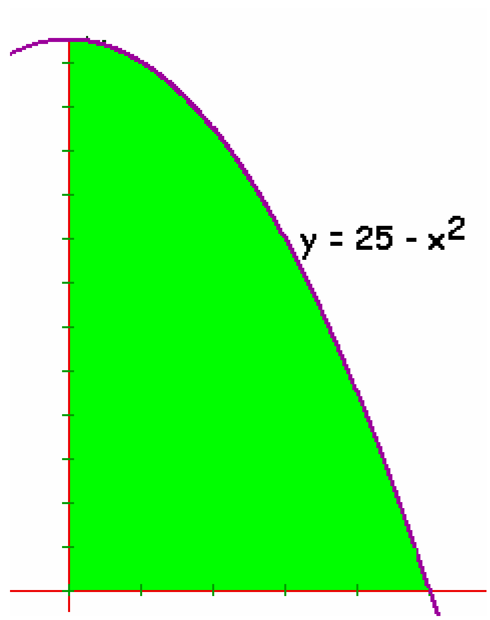
Integrate

$$\int 4\sin^2(3x)dx =$$

$$\int \frac{dx}{\sqrt{9-4x^2}} =$$

$$\int x\sin(4x)\,dx =$$

Find the area of this region:



What is the area under the curve $y = \frac{12}{x+1}$ between x = 2 and x = 3 ?

Use Simpson's Rule with n = 4 to approximate $\int_0^2 \left(x^2 + 1\right)^{0.3} dx =$

Chris spent his break in Cuba, on the beach, in the sun. Life is hard! One day he rolled down a steep slope starting at 3 m/s and accelerating at 2 m/s^2. How fast was he moving 7 seconds later ?

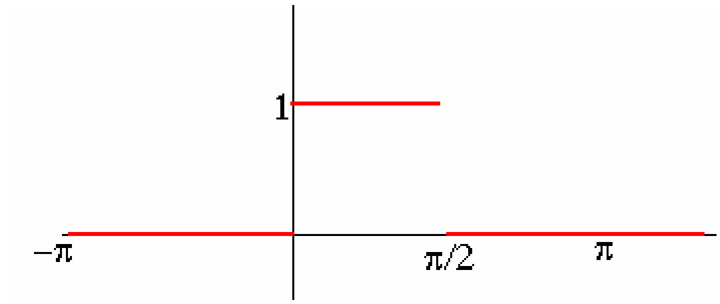
Solve for y: $x^3 y^2 y' = 666$

Solve for y: $y' + \frac{2y}{x} = x$

6 mark question

Consider the function that is 1 for $0 \le x \le \frac{\pi}{2}$ and 0 for the rest of $-\pi$ to π.

Here is its graph:



I am interested in its Fourier Expansion.

What is a_0 =

What is a_1 =

What is b_1 =

Use these values to write the beginning of the Fourier Expansion of the function.

Answers

17.67

yes

$$y' = \frac{3x\cos 3x - \sin 3x}{x^2}$$

$$y' = \frac{\sec^2(\ln 3x)}{x}$$

$$y' = \frac{\sec\sqrt{x}\tan\sqrt{x}}{2\sqrt{x}}$$

$$y' = \frac{1}{1+(x-5)^2}$$

$$y' = 3x^2\ln(x-1) + \frac{x^3}{x-1}$$

$y' = 5(e^x - 2)^4 e^x$

$$y' = \frac{\cos x - y}{\frac{1}{y} + x}$$

$y = -0.8x + 1.1235$

2.4338

4 m by 8 m

$$-\frac{1}{2x^2} + \frac{3x^{4/3}}{4} + C$$

0.0130

$\ln(1 + \tan x) + C$

546.9574

$e^{\sec x} + C$

$2x - 1/3 \sin 6x + C$

$$\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right) + C$$

$$-\frac{x}{4}\cos 4x + \frac{1}{16}\sin 4x + C$$

83.3333

3.4521

2.5094

17 m/s

$$\frac{1}{3}y^3 = \frac{333}{x^2} + C$$

$$y = \frac{x^2}{4} + \frac{C}{x^2}$$

$$a_0 = \frac{1}{4}$$

$$a_1 = \frac{1}{\pi}$$

$$b_1 = \frac{1}{\pi}$$

$$\frac{1}{4} + \frac{1}{\pi}\cos x + \frac{1}{\pi}\sin x + ...$$