(Marks)

1. Evaluate each of the following integrals without the use of integration tables.

(3) (a) $\int \left(\cos x + 5^x + \sqrt{4x} - e^5\right)\, dx$

(3) (b) $\int \frac{x^2 - x + 1}{x - 1}\, dx$

(3) (c) $\int 2x\, \ln(3x)\, dx$

(4) (d) $\int_1^3 \frac{e^{\frac{3}{x}}}{x^2}\, dx$

(4) (e) $\int \frac{x^2 + 1}{e^x}\, dx$

(4) (f) $\int \frac{-2x^2 - 3x - 3}{(x+2)^2(x+1)}\, dx$

(4) (g) $\int_0^{\sqrt{\frac{\pi}{3}}} x \sec(x^2) \tan(x^2)\, dx$

(4) 2. Use the trapezoidal rule with $n = 5$ to approximate $\int_{10}^{20} \frac{\ln x}{x^2}\, dx$. Round your answer to four decimal places.

3. Use the table of integrals to solve each of the following. In each case, state the formula number and justify its use.

(4) (a) $\int \frac{3}{x\sqrt{9 - 4x^2}}\, dx$

(4) (b) $\int \frac{1}{(x+3)^2\sqrt{x^2 + 6x + 13}}\, dx$

(4) 4. Given $f''(x) = 4 - 6x - 40x^3$ with $f'(0) = 4$ and $f(1) = 5$, find $f(x)$.

(4) 5. Find y given the differential equation $(x+1)\frac{dy}{dx} = y$ with condition $y(4) = 10$ and $x > 0$, $y > 0$.

(6) 6. A small town is raising funds to buy a fire engine that costs \$70 000. The initial amount in the fund is \$10 000. It is determined that t months after the beginning of the fund raiser, the rate at which money is contributed to the fund is proportional to the difference between the desired goal of \$70 000 and the total amount F in the fund at that time. After 1 month a total of \$40 000 is in the fund.

(a) Write the differential equation with initial conditions.

(b) Solve the differential equation for F as a function of t.

(c) How much will be in the fund after 3 months?

(4) 7. Given the curves $f(x) = x^2$ and $g(x) = 4x - x^2$, determine

(a) the point(s) of intersection of $f(x)$ and $g(x)$,

(b) the area bounded by $f(x)$ and $g(x)$.

(6) 8. Given the demand function $p = \frac{100}{\sqrt{2x + 50}}$ and the supply function $p = \sqrt{2x + 50}$,

(a) find the equilibrium point,

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(b) sketch and identify the regions representing the consumer and producer surpluses,

(c) evaluate the consumer surplus.

(6) 9. Use l'Hôpital's rule to evaluate the following limits

(a) $\lim_{x \to 0} \frac{\cos x - 1}{x^2}$

(b) $\lim_{x \to \infty} \frac{x}{\ln(x + e^x)}$

(8) 10. Evaluate each improper integral and state whether it converges or diverges

(a) $\int_1^6 \frac{1}{(6-x)^3}\, dx$

(b) $\int_{e^2}^{\infty} \frac{1}{x(\ln x)^2}\, dx$

(3) 11. Consider the sequence $\left\{ \frac{1}{3}, \frac{-2}{6}, \frac{-5}{9}, \frac{-8}{12}, \ldots \right\}$

(a) Give the next three terms of the sequence.

(b) Give an expression for the n^{th} term of the sequence.

(6) 12. Determine the convergence or divergence of each sequence $\{a_n\}$. If the sequence converges, find the limit.

(a) $a_n = \frac{2n!}{(n-1)!}$

(b) $a_n = \frac{(2+n)^2}{4 - 3n - 2n^2}$

(6) 13. Determine with *justification* if each of the following series is convergent or divergent. If the series is convergent, *find its sum*.

(a) $\sum_{n=1}^{\infty} \frac{\sqrt{1+4n^2}}{6n}$

(b) $\sum_{n=1}^{\infty} \left(\frac{3^n}{4^{n-1}} + \frac{1}{4^n} \right)$

(3) 14. Given a repeated decimal $3.0\overline{7}$, express it using a geometric series, find the sum of the geometric series and write the decimal as the ratio of two integers.

(3) 15. An amount of \$3000 is deposited in a bank that pays 3% interest, compounded monthly. Find the balance at the end of 10 years.

(4) 16. A deposit of \$60 is made at the beginning of each quarter into a savings account that pays 2% interest a year, compounded quarterly. Find the balance after 4 years.

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ANSWERS

(1 a) $\sin x + \frac{5^x}{\ln(5)} + \frac{4x^{3/2}}{3} - e^5 x + C$; (1 b) $\frac{x^2}{2} + \ln|x-1| + C$; (1 c) $x^2 \ln(3x) - \frac{x^2}{2} + C$

(1 d) $\frac{e^3}{3} - \frac{e}{3} \approx 5.79$; (1 e) $-(x^2+1)e^{-x} - 2xe^{-x} - 2e^{-x} + C$; (1 f) $\frac{-5}{x+2} - 2\ln|x+1| + C$; (1 g) $\frac{1}{2}$

(2) 0.1315 ; (3 a) F19 $\implies -\ln\left|\frac{3+\sqrt{9-4x^2}}{2x}\right| + C$; (3 b) F29 $\implies -\frac{\sqrt{x^2+6x+13}}{4(x+3)} + C$

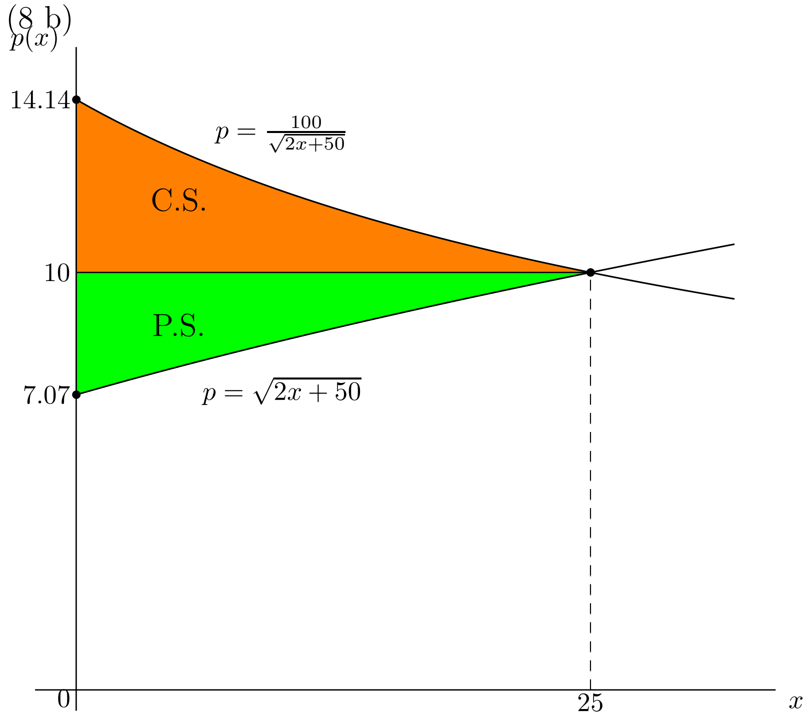
(4) $f(x) = 2 + 4x + 2x^2 - x^3 - 2x^5$; (5) $y = 2(x+1)$

(6 a) $\frac{dF}{dt} = k(70000 - F)$ with conditions: $F(0) = 10000$; $F(1) = 40000$; (6 b) $F(t) = 70000 - 60000e^{-\ln(2).t}$

(6 c) \$62500 ; (7 a) $(0,0)$ and $(2,4)$; (7 b) $\frac{8}{3} \approx 2.66$ square units

(8 a) $x_e = 25$ and $p_e = 10$

(8 b)



(8 c) \$42.89

(9 a) $-\frac{1}{2}$; (9 b) 1

(10 a) diverges

(10 b) converges to $\frac{1}{2}$

(11 a) $\frac{-11}{15}, \frac{-14}{18}, \frac{-17}{21}$

(11 b) $a_n = \frac{4-3n}{3n}$; (12 a) sequence diverges ; (12 b) sequence converges to $-\frac{1}{2}$; (13 a) series diverges

(13 b) series converges and has sum $\frac{37}{3}$; (14) $\frac{277}{90}$; (15) \$4048.06 ; (16) \$1001.84