

1. Evaluate the following limits:

$$(a) \lim_{x \rightarrow \infty} \frac{5x^3 - 7x + 12}{7x - 4x^3} \quad (b) \lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 5} - 3}{x - 2}$$

$$(c) \lim_{x \rightarrow 0} \frac{\sin(4x)}{\tan(3x)} \quad (d) \lim_{x \rightarrow 1^+} \left(\frac{1}{x-1} + \frac{1}{x^2 - 3x + 2} \right)$$

$$(e) \lim_{x \rightarrow 1} f(x) \text{ if } 2x - 1 \leq f(x) \leq x^2 \text{ for } 0 < x < 3$$

2. The function $f(x)$ is defined by:

$$f(x) = \begin{cases} x^2 + 3 & \text{if } x \leq -2 \\ -5x & \text{if } -2 < x < 5 \\ \frac{x^2}{x-6} & \text{if } x \geq 5 \end{cases}$$

(a) Give all values of x where $f(x)$ is not continuous. State the type of discontinuity at each of these values and justify your answer.

(b) At what values of x is $f(x)$ continuous but not differentiable?

3. Given $f(x) = 3 - \frac{2}{x+1}$.

(a) Find and simplify $\frac{f(2+h) - f(2)}{h}$.

(b) Use the result of part (a) to find $f'(2)$.

(c) Use the derivative rules to check your answer to part (b).

(d) Find the equation of the normal line to the curve at $x = 2$.

4. Find $\frac{dy}{dx}$ for each of the following:

$$(a) y = (2x - 5)^4(x^2 + 3)^3 \quad (b) y = \frac{3}{x^4} - \sqrt[4]{x^3} + 4 \log_3 x + 3^{4x} - 4e^3$$

$$(c) x^2 y^2 + x \sin y = 4 \cos(3x) \quad (d) y = \ln \left(\frac{x^2 \sqrt{\sin x}}{(2x - 3)^3} \right)^4$$

$$(e) y = (2x - 1)^{\tan(3x)}$$

5. For what value(s) of x will $f(x) = \frac{e^{2x}}{x-3}$ have a horizontal tangent line?

6. Find the critical numbers for $f(x) = \ln(x^4 - 8x^2 + 17)$

7. Find the absolute extrema for $f(x) = \frac{\sin x}{2 + \cos x}$ on the interval $[0, \pi]$.

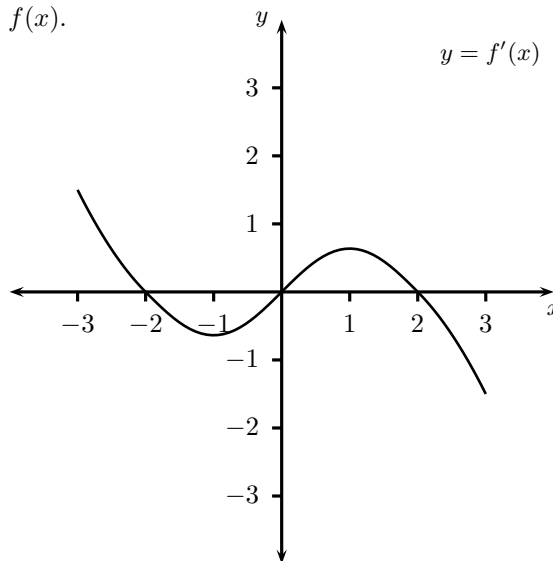
8. If $f(x) = \frac{e^{2x}}{e^{2x} - 3}$, give the equations of all vertical and horizontal asymptotes of $f(x)$.

9. Given $f(x) = \frac{1}{3}x^{5/3}(8-x)$, $f'(x) = \frac{8}{9}x^{2/3}(5-x)$ and $f''(x) = \frac{40(2-x)}{27x^{1/3}}$

Sketch the graph of $f(x)$ clearly showing all (if any) asymptotes, intercepts, local extrema and points of inflection.

10. In this question we are interested in an unknown function $f(x)$.

The figure on the right shows the graph of $f'(x)$, the derivative of the unknown function, on the interval $[-3, 3]$.



- (a) Sketch the graph of $f''(x)$
- (b) Use these two graphs to find:
 - (i) the interval(s) where $f(x)$ is increasing
 - (ii) the interval(s) where $f(x)$ is concave down
 - (iii) the critical values of $f(x)$
 - (iv) the x -coordinates of the point(s) of inflection of $f(x)$

11. Sand is poured into a conical pile with the height of the pile always equalling its diameter. If the sand is poured at the constant rate of $5 \text{ m}^3/\text{s}$, at what rate is the height of the pile changing when the height is 2 meters? [$V = \frac{1}{3}\pi r^2 h$]

12. A cylindrical package to be sent by a postal service can have a maximum combined length and girth (perimeter of the circular cross section) of 84 inches. Find the dimensions of the package with the maximum volume.

13. The function $f(x)$ is defined by $f(x) = x^3 + 3x - 2$.

- (a) Use the Intermediate Value Theorem to show that the equation $f(x) = 0$ has at least one real root.
- (b) Show that the equation $f(x) = 0$ has exactly one real root.

14. Evaluate the following:

(a) $\int_1^2 \left(x + \frac{1}{x^2}\right)^2 dx$ (b) $\int \left(3x^7 - \sqrt{x^7} + \frac{3^x}{7} - e^3\right) dx$

(c) $\int \left(\frac{4 \sin \theta}{\cos^2 \theta}\right) d\theta$ (d) $\int \frac{(\sqrt{t} + 3)(\sqrt{t} - 3)}{3t} dt$

15. Use both parts of the Fundamental Theorem of Calculus to find:

$$\frac{d}{dx} \left(\int_{\pi/2}^{x^2} \frac{\sin t}{t} dt \right) - \int_{\pi/2}^{x^2} \frac{d}{dt} \left(\frac{\sin t}{t} \right) dt$$

16. Given

$$f(x) = \begin{cases} |x+2| & \text{if } x \leq 0 \\ \sqrt{4-x^2} & \text{if } 0 < x \leq 2 \end{cases}$$

Evaluate $\int_{-5}^2 f(x) dx$ by interpreting it in terms of areas.

17. Find the area under the curve $f(x) = 2 - \cos x$ and above the x -axis from $x = \frac{\pi}{6}$ to $x = \pi$.

18. Find an approximation for $\int_2^{10} (x^2 - 3x) dx$ using a Riemann sum with right endpoints

and dividing the interval into 4 equal parts (i.e. using 4 rectangles.)

ANSWERS

1. (a) $-5/4$ (b) $2/3$ (c) $4/3$ (d) -1 (e) 1

2. (a)

- $x = -2$, jump discontinuity; one-sided limits at -2 do not equal each other.
- $x = 6$, infinite discontinuity; right-sided limit at 6 equals ∞ .

(b) $x = 5$

3. (a) $\frac{2}{3(3+h)}$ (b) $f'(2) = \lim_{h \rightarrow 0} \frac{2}{3(3+h)} = \frac{2}{9}$

(c) $f'(x) = \frac{2}{(x+1)^2}$, thus $f'(2) = \frac{2}{9}$ (d) $y = -\frac{9}{2}x + \frac{34}{3}$

4. (a) $2(2x-5)^3(x^2+3)^2(10x^2-15x+12)$

(b) $-\frac{12}{x^5} - \frac{3}{4x^{1/4}} + \frac{4}{(\ln 3)x} + 4(\ln 3)3^{4x}$

(c) $\frac{-12 \sin(3x) - \sin(y) - 2xy^2}{2x^2y + x \cos y}$

(d) $\frac{8}{x} + \frac{2 \cos x}{\sin x} - \frac{24}{2x-3}$

(e) $y \left[3 \sec^2(3x) \ln(2x-1) + \frac{2 \tan(3x)}{2x-1} \right]$

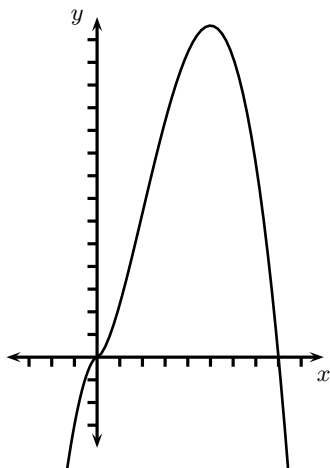
5. $7/2$

6. $-2, 0, 2$

7. Absolute maximum is $1/\sqrt{3}$, absolute minimum is 0 .

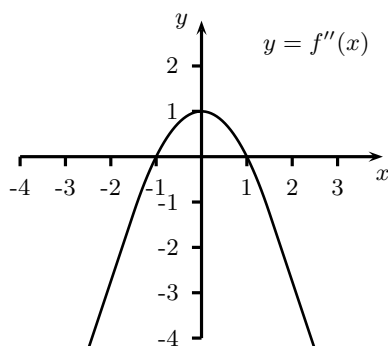
8. Horizontal asymptotes: $y = 0$, $y = 1$. Vertical asymptote: $x = \ln \sqrt{3}$.

9.



- x -intercepts : $(0, 0)$, $(8, 0)$
- y -intercept: $(0, 0)$
- Absolute maximum of $5\sqrt[3]{25}$ occurs at $x = 5$.
- Inflection points: $(0, 0)$, $(2, 4\sqrt[3]{4})$

10. (a)



- (b)
- (i) $(-3, -2)$, $(0, 2)$
 - (ii) $(-3, -1)$, $(1, 3)$
 - (iii) $-2, 0, 2$
 - (iv) $-1, 1$

11. $5/\pi$ m/s

12. Length: 28 inches, girth: 56 inches.

13. (a) Apply the Intermediate Value Theorem with $f(x)$ and the interval $[0, 1]$. (Details omitted).

(b) Note that $f'(x) = 3x^2 + 3 > 0$ for all x , thus $f(x)$ is always increasing. This implies $f(x) = 0$ has only the one solution.

14. (a) $\frac{21}{8} + 2 \ln 2$ (b) $\frac{3}{8}x^8 - \frac{2}{9}\sqrt{x^9} + \frac{3^x}{7 \ln 3} - e^3 x + C$

(c) $4 \sec \theta + C$ (d) $\frac{t}{3} - 3 \ln |t| + C$

15. $\frac{\sin(x^2)}{x^2}(2x - 1) + \frac{2}{\pi}$

16. $\pi + \frac{13}{2}$

17. $\frac{5}{3}\pi + \frac{1}{2}$

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