(3) 1. Evaluate the expressions:

(a) ${}_{10}C_4$ (b) ${}_{10}P_4$ (c) $\dfrac{15!4!}{3!11!}$

(4) 2. (a) In how many ways can a president, a vice president and a treasurer be selected for a club with 7 members?

(b) Six students are going to be chosen at random from a class of 20 students to take a trip to Ottawa. In how many ways can this be done?

(3) 3. Let $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $U = \{x \in N : 1 \leq x \leq 10\}$.

a) Are A and B equivalent? Explain.

b) Find $\overline{A}$.

c) Find $A \cap B$ and $A \cup B$

(2) 4. Draw a Venn diagram and show by hatching the following set: $(B \cap \overline{C}) \cup (\overline{A \cup C})$

(5) 5. For each expression, name the property from the given list and say whether it is a set property, a network property, or a logic property: *associative, closure, commutative, complement, contradiction, DeMorgan, distributive, double complement, double negation, idempotent, identity, property of 1 (or 0), tautology.*

a) $(p \wedge p) \leftrightarrow p$ b) $A \cap \overline{A} = \emptyset$ c) $\overline{A \cdot B} = \overline{A} + \overline{B}$
d) $\sim (\sim p) \leftrightarrow p$ e) $A + B \cdot C = (A + B) \cdot (A + C)$

(1) 6. What is a contradiction?

(4) 7. Use a truth table to determine whether or not $\overline{\wedge}$ (NAND) distribute over $\underline{\vee}$ (EOR). That is, whether or not $[p \,\overline{\wedge}\, (q \,\underline{\vee}\, r)]$ is equivalent to $[(p \,\overline{\wedge}\, q) \,\underline{\vee}\, (p \,\overline{\wedge}\, r)]$.

(6) 8. Use a truth table to determine whether the argument is valid or not.

H: If there are two books on the table, then one of them belongs to Jane.
If one of the books belongs to Jane, then Jane is happy.
Jane is not happy.

C: There are not two books on the table.

(3) 9. Use a Venn diagram to determine the validity of the argument.

H: All Pandas are black and white.
Some black and white animals are endangered.

C: Some Pandas are endangered.

(4) 10. If you have a flu, then you must stay home for seven days.

a) Write the converse, the contrapositive, and the inverse of the implication above.
b) Say which among the four statements are equivalent.

(4) 11. Find a Boolean table for the expression:

a) $A \cdot B + \overline{A} \cdot \overline{B}$

b) Write a logical statement which is analogous to this expression.

(2) 12. Draw a network to represent the given boolean expression:

$F[(A + CD) + (\overline{E}FG)]$

(5) 13. Simplify each expression, justifying each step using properties of Boolean algebra.

a) $A + \overline{A}B + \overline{B}$

b) $AB + BC + B(\overline{C} + \overline{B})$

(6) 14. Classify each system below (*without solving*) as dependent or independent, consistent with one solution only, consistent with infinitely many solutions or inconsistent. Justify your answers.

a) $\begin{array}{rcl} 3x - 2y & = & 3 \\ 2x - 3y & = & 3 \end{array}$

b) $\begin{array}{rcl} 5x - 2y & = & 3 \\ 2y - 5x & = & -3 \end{array}$

c) $\begin{array}{rcl} 3x - y & = & 6 \\ -6x + 2y & = & -3 \end{array}$

(5) 15. Given the system $\begin{cases} 2x - 3y = 7 \\ -x + 3y = -2 \end{cases}$

a) Estimate the solution by graphing.

b) Solve the system algebraically by substitution or elimination.

c) Compare results from a) and b) (Write a comment).

(12) 16. Given: $A = \begin{bmatrix} 0 & -2 \\ -1 & 3 \\ 5 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & 7 \end{bmatrix}$

$D = \begin{bmatrix} 1 & -1 & 3 \\ -6 & 2 & 10 \\ 3 & 0 & 5 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

find each of the following, if possible. If an operation is not possible, say why.

a) $3B - 5I$ b) C^{-1} c) B^4

d) $C + I$ e) AI f) $A^T D$

g) CD

(10) 17. Find the inverse of each matrix or explain why it does not exist.

a) $A = \begin{bmatrix} 1 & 0 \\ 3 & 3 \end{bmatrix}$ b) $B = \begin{bmatrix} 1 & -2 & 4 \\ 2 & -5 & 6 \\ -3 & 9 & -8 \end{bmatrix}$ c) $C = \begin{bmatrix} 2 & -3 & 4 \\ 2 & 5 & -3 \\ 0 & -8 & 7 \end{bmatrix}$,

(4) 18. Given the linear system $\begin{cases} x - y - 3z = 0 \\ -x + 2y + 4z = -1 \\ x + y = 3 \end{cases}$

a) write the system in matrix form $AX = B$.

b) If $A^{-1} = \begin{bmatrix} -4 & -3 & 2 \\ 4 & 3 & -1 \\ -3 & -2 & 1 \end{bmatrix}$, solve the system using A^{-1}.

(4) 19. Given a linear system $AX = B$.

a) If the reduced form of augmented matrix is $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 1 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$ what is the solution, if any?

b) If the augmented matrix has a form $\left[\begin{array}{ccc|c} 1 & 0 & 5 & -5 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right]$ what is the solution, if any?

(6) 20. If possible solve the following system using matrices and row operations.

$$\begin{aligned} 3x + 8y - 4z &= -6 \\ x + 3y - 2z &= -2 \\ -x + 4y - 7z &= 7 \end{aligned}$$

(4) 21. If possible, solve the following system using matrices and row operations.

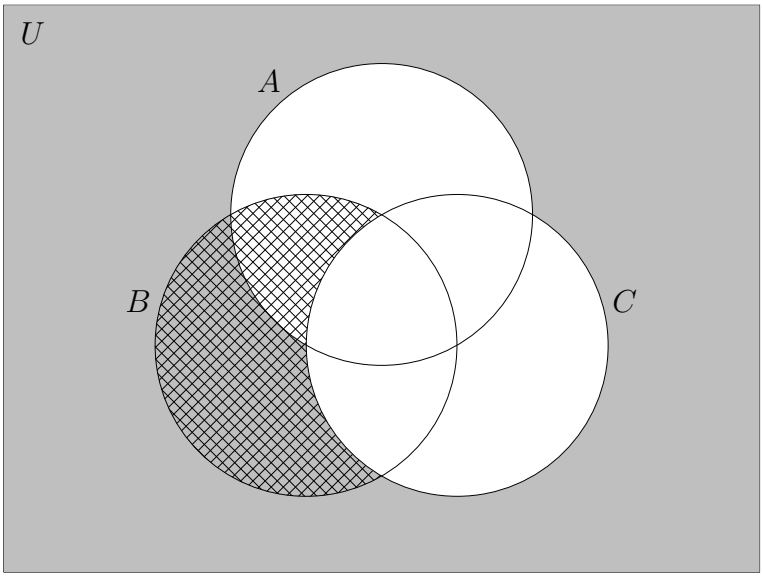
$$\begin{aligned} 2x - y + 5z &= 5 \\ 3x + 2y - z &= 3 \\ 4x - 2y + 10z &= 9 \end{aligned}$$

(3) 22. Prove by mathematical induction that for all positive integers n

$$2 + 4 + 8 + \cdots + 2^n = 2(2^n - 1) \qquad \text{[Note: } 2^{m+n} = 2^m 2^n \text{]}$$

ANSWERS:

1. a) 210 b) 5040 c) 131040
2. a) ${}^7P_3 = 210$ b) ${}^{20}C_6 = 38760$
3. a) Yes, A and B have the same number of elements.
   b) $\overline{A} = \{2, 4, 6, 7, 8, 9, 10\}$
   c) $A \cap B = \emptyset$ and $A \cup B = \{1, 2, 3, 4, 5, 6\}$
4. Solution is anything shaded or hatched



5. a) idempotent, logic b) complement, sets c) DeMorgan, network
   d) double negation, logic e) distributive, network

6. Logical statement which is false in every case.

7. Not equivalent

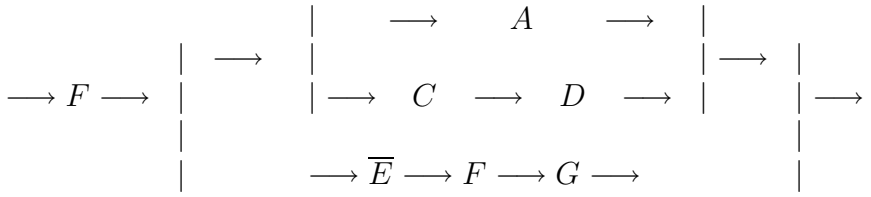
8. Valid argument

9. Invalid argument

10. a) Converse: If you must stay home for seven days, then you have a flu.
    Contrapositive: If you don't have to stay home for seven days, then you don't have a flu.
    Inverse: If you don't have a flu, then you don't have to stay home for seven days.

    b) The original statement is equivalent to contrapositive. The converse statement is equivalent to inverse.

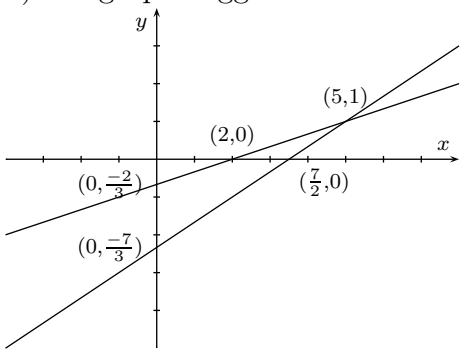
11. a) It is 0 when $A \neq B, (A = 1, B = 0$ or $A = 0, B = 1)$ and 1 when $A = B (A = 1, B = 1$ or $A = 0, B = 0)$.
    b) $p \leftrightarrow q$ (or equivalent statement $(p \wedge q) \vee (\sim p \wedge \sim q)$)

12. 

13. a) 1 b) B

14. a) different slopes, independent and consistent, one solution
    b) same line, dependent and consistent, infinitely many solutions
    c) same slope and different intercepts means parallel lines, independent and inconsistent system, no solution.

15. a)The graph suggests that $x \simeq 5$ and $y \simeq 1$, which turns out to be the exact answer by b).



16. a) $\begin{bmatrix} 7 & -9 \\ -3 & 1 \end{bmatrix}$ b) no inverse, C is not square matrix c) $\begin{bmatrix} 469 & -468 \\ -156 & 157 \end{bmatrix}$
    d) undefined e) $\begin{bmatrix} 0 & -2 \\ -1 & 3 \\ 5 & 1 \end{bmatrix}$ f) $\begin{bmatrix} 21 & -2 & 15 \\ -17 & 8 & 29 \end{bmatrix}$ g) $\begin{bmatrix} 25 & -7 & -17 \\ 15 & 2 & 45 \end{bmatrix}$

17. a) $\begin{bmatrix} 1 & 0 \\ -1 & \frac{1}{3} \end{bmatrix}$ b) $\begin{bmatrix} -7 & 10 & 4 \\ -1 & 2 & 1 \\ \frac{3}{2} & -\frac{3}{2} & -\frac{1}{2} \end{bmatrix}$ c) No inverse, C can not be reduced to I.

18. a) $\begin{bmatrix} 1 & -1 & -3 \\ -1 & 2 & 4 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}$ b) $X = A^{-1}B = \begin{bmatrix} 9 \\ -6 \\ 5 \end{bmatrix}$

19. a) $\begin{aligned} x &= 1 - 3t \\ y &= 5 + 2t \\ z &= t \end{aligned}$ b) $\begin{aligned} x &= -5 \\ y &= 3 \\ z &= 0 \end{aligned}$

20. $\begin{aligned} x &= -6 \\ y &= 2 \\ z &= 1 \end{aligned}$

21. System inconsistent (no solution).

22.