

Mathematical Models 1

201-115

Fall 2009

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Final Examination

Please give numeric answers to 4 decimal places, except for accuracy/precision/significant digits questions.

1 mark questions

Calculate to the right accuracy:

$$\frac{16.03(0.0025)}{2.3} =$$

$$1.94 + 13.53 + 6.092 =$$

Write 0.66 rads in degrees

Calculate

$$e^{2.1} =$$

$$\ln 0.0041 =$$

$$\ln -12 =$$

The waves in my waterbed have an amplitude of 5 cm and a frequency of 0.08 Hz. What is their Angular Velocity ?

2 mark questions

Write using simple logs

$$\ln\left(\frac{7x+3}{x^7}\right) =$$

Solve for x

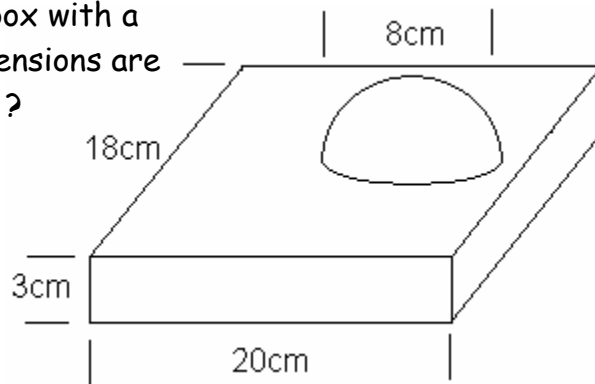
$$5(3^{x+2}) = 10\,000$$

Find x :

$$\log x + \log(x + 1) = \log 90$$

A triangular sail, 4 m high and 1.5 m along the base, faces a wind machine some 20 m away. What solid angle (in steradians) does the sail make at the wind machine ?

This diagram tries to show a 3-d box with a bump on one side. Some of its dimensions are marked. What is its Surface Area ?



Solve for x and y :

$$\begin{aligned} 3x - 2y &= 26 \\ 4x + 5y &= 27 \end{aligned}$$

Set up the determinants to find y , but don't bother to work it out:

$$2x + 6y - 3z = 26$$

$$3x - 4y + 3z = -15$$

$$4x + 5y + 6z = 7$$

Calculate:

$$\begin{vmatrix} 3 & 0 & 6 \\ 1 & 8 & 2 \\ -2 & 4 & 1 \end{vmatrix} =$$

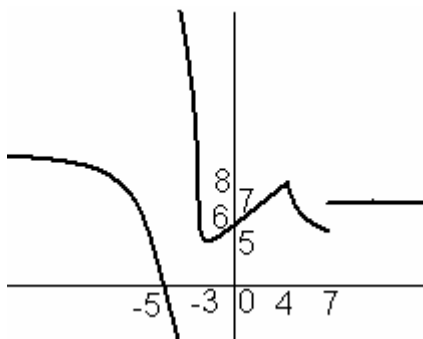
$$\overrightarrow{(4,-3)} + \overrightarrow{(-8,7)} =$$

A butterfly is trying to fly North at 0.7 m/s. But a gentle breeze pushes it West at 1.3 m/s. What is the magnitude and direction of its motion?

Find the limits

$$\lim_{x \rightarrow \infty} \frac{e^x - 1}{x} =$$

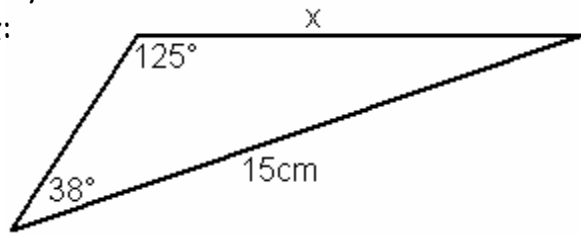
$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 5x + 6} =$$



Here is the graph of a function. Is it continuous everywhere? If not, indicate where it is not continuous.

3 mark questions

Find x :



The Town of Ste Anne's is thinking of putting a park on a triangle of land with sides of 75m , 56m and 48m . What is the angle at the smallest corner of the land?

Sketch the graph of $y = 12 \sin(8t + \pi)$

Vertical Shift =

Amplitude =

Phase Shift =

Period =

Solve for all possible values of X , an angle between 0 and 2π radians:

$$3 \tan X + 4 = \tan X$$

Calculate

$$(3+5j)(2-11j) =$$

$$\frac{12-5j}{2+3j} =$$

Write $17 \text{ cis } 40^\circ$ in rectangular form.

Write $17 \text{ cis } 40^\circ$ in exponential form.

Calculate:

$$(12 \text{ cis } 45^\circ) (3 \text{ cis } 55^\circ) =$$

$$(1.2 \text{ cis } 15^\circ)^5 =$$

Find the fourth roots of $81 \text{ cis } 60^\circ$

A 50Ω resistor is in series with a $200 \mu\text{F}$ capacitor. If a 60Hz current flows through them, what is their Impedance ?

Find the derivatives:

$$y = \frac{5}{x^2} - \frac{x^2}{5}$$

$$y = 3x \sin(2x)$$

$$y = \frac{3x^2 - 1}{2x + 5}$$

$$y = (6x - 7)^8 + 9$$

$$y = 3 \cos(\sqrt{x})$$

$$y = \sqrt{4 - \tan 2x}$$

Find the equation of the line tangent to $y = 3x^3 - 3x$ at the point where $x = 2$

Find the slope of this curve at the point (1, 2):

$$7x + 5y = x^2y + 15$$

What is the second derivative of $y = \frac{4}{3x-1}$?

4 mark question

Use the Limit Definition of Derivative to find the derivative of $y = 5x^2 - x + 4$

Show all the steps.

Answers

0.017 (2 sig digs)

21.56

37.81°

8.1661

-5.4968

sfa

0.5026

$\ln(7x + 3) - 7 \ln x$

4.9186

9 only

0.0075 steradians

998.26 cm²

$x = 8$ and $y = -1$

$$y = \frac{\begin{vmatrix} 2 & 26 & -3 \\ 3 & -15 & 3 \\ 4 & 7 & 6 \end{vmatrix}}{\begin{vmatrix} 2 & 6 & -3 \\ 3 & -4 & 3 \\ 4 & 5 & 6 \end{vmatrix}}$$

120

(-4, 4)

1.4765 m/s at 28.28° North of West

infinity

6

Not continuous; circle breaks at -5 and 7

11.27 cm

39.76°

sine wave with $V_S = 0$

Amp = 12

PS = -0.39

Period = 0.785

116.57° and 296.57°

61 - 23j

$$(9 - 46j) / 13$$

$$13.02 + 10.92j$$

$$17e^{0.6981j}$$

$$36 \text{ cis } 100^\circ$$

$$2.4883 \text{ cis } 75^\circ$$

$$3 \text{ cis } 15^\circ, 3 \text{ cis } 105^\circ, 3 \text{ cis } 195^\circ, 3 \text{ cis } 285^\circ$$

$$50 - 13.26j \Omega$$

$$-10x^{-3} - 2/5x$$

$$3 \sin 2x + 6x \cos 2x$$

$$(6x^2 + 30x + 2) / (2x + 5)^2$$

$$48(6x - 7)^7$$

$$\frac{3 \sin \sqrt{x}}{2\sqrt{x}}$$

$$\frac{\sec^2 2x}{\sqrt{4 - \tan 2x}}$$

$$y = 33x - 48$$

$$-0.75$$

$$y'' = 72(3x - 1)^{-3}$$

$$y' = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(5(x+h)^2 - (x+h) + 4) - (5x^2 - x + 4)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{5x^2 + 10xh + 5h^2 - x - h + 4 - 5x^2 + x - 4}{h}$$

$$= \lim_{h \rightarrow 0} \frac{10xh + 5h^2 - h}{h}$$

$$= \lim_{h \rightarrow 0} 10x + 5h - 1$$

$$= 10x - 1$$