Mathematical Models 1

201-115
Fall 2009
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Final Examination

Please give numeric answers to 4 decimal places, except for accuracy/precision/significant digits questions.

1 mark questions

Calculate to the right accuracy:

$$\frac{16.03(0.0025)}{2.3} =$$

1.94 + 13.53 + 6.092 =

Write 0.66 rads in degrees

Calculate

$e^{2.1}$ =

ln 0.0041 =

ln -12 =

The waves in my waterbed have an amplitude of 5 cm and a frequency of 0.08 Hz. What is their Angular Velocity ?

2 mark questions

Write using simple logs

$$\ln\left(\frac{7x+3}{x^7}\right) =$$

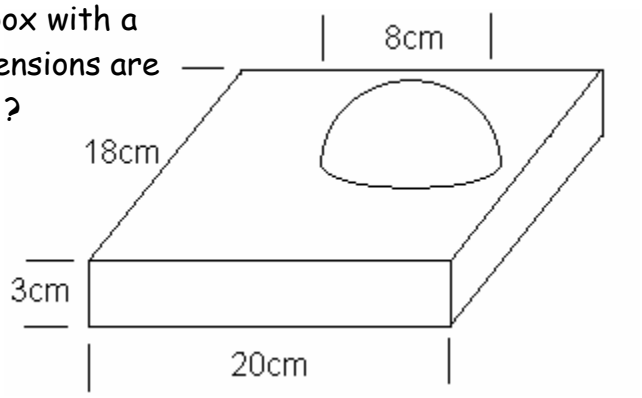
Solve for x

$5\,(3^{x+2}) = 10\,000$

Find x :
$\log x + \log(x + 1) = \log 90$

A triangular sail, 4 m high and 1.5 m along the base, faces a wind machine some 20 m away. What solid angle (in steradians) does the sail make at the wind machine ?

This diagram tries to show a 3-d box with a bump on one side. Some of its dimensions are marked. What is its Surface Area ?



Solve for x and y: $3x - 2y = 26$
$4x + 5y = 27$

Set up the determinants to find y, but don't bother to work it out:
$2x + 6y - 3z = 26$
$3x - 4y + 3z = -15$
$4x + 5y + 6z = 7$

Calculate:

$$\begin{vmatrix} 3 & 0 & 6 \\ 1 & 8 & 2 \\ -2 & 4 & 1 \end{vmatrix} =$$

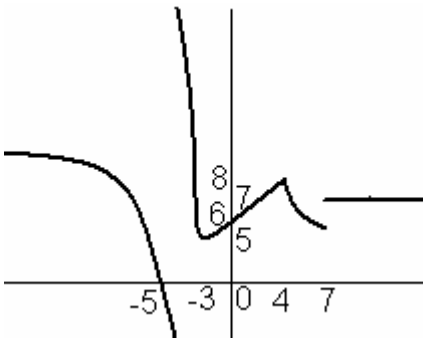
$$\overrightarrow{(4,-3)} + \overrightarrow{(-8,7)} =$$

A butterfly is trying to fly North at 0.7 m/s. But a gentle breeze pushes it West at 1.3 m/s. What is the magnitude and direction of its motion?

Find the limits

$$\lim_{x \to \infty} \frac{e^x - 1}{x} =$$

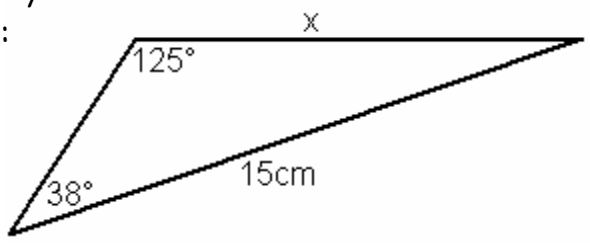
$$\lim_{x \to 3} \frac{x^2 - 9}{x^2 - 5x + 6} =$$



Here is the graph of a function. Is it continuous everywhere ? If not, indicate where it is not continuous.

3 mark questions

Find x:



The Town of Ste Anne's is thinking of putting a park on a triangle of land with sides of 75m, 56m and 48m. What is the angle at the smallest corner of the land?

Sketch the graph of $y = 12 \sin(8t + \pi)$

Vertical Shift =

Amplitude =

Phase Shift =

Period =

Solve for all possible values of X, an angle between 0 and 2π radians:

$3 \tan X + 4 = \tan X$

Calculate
$(3+5j)(2-11j) =$

$\frac{12-5j}{2+3j} =$

Write 17 cis 40° in rectangular form.

Write 17 cis 40° in exponential form.

Calculate:
(12 cis 45°) (3 cis 55°) =

$(1.2 \text{ cis } 15°)^5 =$

Find the fourth roots of 81 cis 60°

A 50Ω resistor is in series with a 200 μFd capacitor. If a 60Hz current flows through them, what is their Impedance ?

Find the derivatives:

$$y = \frac{5}{x^2} - \frac{x^2}{5}$$

$$y = 3x\sin(2x)$$

$$y = \frac{3x^2 - 1}{2x + 5}$$

$$y = (6x - 7)^8 + 9$$

$$y = 3\cos(\sqrt{x})$$

$$y = \sqrt{4 - \tan 2x}$$

Find the equation of the line tangent to $y = 3x^3 - 3x$ at the point where $x = 2$

Find the slope of this curve at the point (1, 2):
$7x + 5y = x^2y + 15$

What is the second derivative of $y = \frac{4}{3x-1}$?

4 mark question
Use the Limit Definition of Derivative to find the derivative of $y = 5x^2 - x + 4$
Show all the steps.

Answers

0.017 (2 sig digs)
21.56
37.81°
8.1661
-5.4968
sfa
0.5026
ln(7x + 3) – 7 ln x
4.9186

9 only
0.0075 steradians
998.26 cm^2
x = 8 and y = -1

$$y = \frac{\begin{vmatrix} 2 & 26 & -3 \\ 3 & -15 & 3 \\ 4 & 7 & 6 \end{vmatrix}}{\begin{vmatrix} 2 & 6 & -3 \\ 3 & -4 & 3 \\ 4 & 5 & 6 \end{vmatrix}}$$

120
(-4, 4)
1.4765 m/s at 28.28° North of West
infinity
6
Not continuous; circle breaks at -5 and 7

11.27 cm
39.76°
sine wave with VS = 0
Amp = 12
PS = -0.39
Period = 0.785
116.57° and 296.57°

61 – 23j

(9 – 46j) / 13

13.02 + 10.92j

$17e^{0.6981j}$

36 cis 100°

2.4883 cis 75°

3 cis 15°, 3 cis 105°, 3 cis 195°, 3 cis 285°

50 – 13.26j Ω

$-10x^{-3} - 2/5\,x$

$3\sin 2x + 6x\cos 2x$

$(6x^2 + 30x + 2) / (2x + 5)^2$

$48(6x - 7)^7$

$$-\frac{3\sin\sqrt{x}}{2\sqrt{x}}$$

$$-\frac{\sec^2 2x}{\sqrt{4-\tan 2x}}$$

y = 33x – 48

-0.75

$y'' = 72(3x - 1)^{-3}$

$$y' = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\left(5(x+h)^2-(x+h)+4\right)-(5x^2-x+4)}{h}$$

$$= \lim_{h\to 0}\frac{5x^2+10xh+5h^2-x-h+4-5x^2+x-4}{h}$$

$$= \lim_{h\to 0}\frac{10xh+5h^2-h}{h}$$

$$= \lim_{h\to 0}10x+5h-1$$

$$= 10x-1$$