(8) 1. Solve each of the following systems or show that it is inconsistent.

a) $\begin{aligned} x + 2y - 5z &= 1 \\ 3x + 7z &= 3 \\ -2x + 5y - 23z &= 3 \end{aligned}$ b) $\begin{aligned} w - x + 2y + 3z &= 2 \\ 3x + 5y + z &= -1 \\ 2w + x - 2y + 3z &= 1 \end{aligned}$.

(5) 2. Given the system $\begin{aligned} x + 2y + z &= 3 \\ 2x + 5y + 3z &= 10 \\ 3x + 7y + (k^2 + 3)z &= k + 14 \end{aligned}$

find the value(s) for k such that the system will have:
a) infinitely many solutions b) no solution c) a unique solution

(6) 3. Cake Plus is a bakery which specializes in three types of cakes: Apple Cake, Chocolate Mousse Cake, and Strawberry Cheesecake. The Apple Cakes sell for \$12 each, the Chocolate Mousse Cakes sell for \$14 each, and the Strawberry Cheesecakes sell for \$16 each. One day Cake Plus sold 20 cakes and revenue was \$300.

a) Define all necessary variables x, y, and z and set up the system of equations required to solve the problem.
b) Find a general (parametric) solution for the system.
c) Find two particular solutions which are realistic.

(7) 4. Given $A^{-1} = \begin{bmatrix} -2 & 4 \\ 8 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 3 & -1 & 4 \\ 0 & 2 & -3 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 5 \\ 7 & -1 \\ 0 & 3 \end{bmatrix}$

find, if possible:
a) AB b) $BC - 2B$ c) DA^{-1} d) $C - 3D^t$

(6) 5. Let a simple economy have two industries: Yellow and Orange. The production of 1\$ of Yellow requires 20¢ of Yellow and 10¢ of Orange. The production of 1\$ of Orange requires 40¢ of Yellow and 70¢ of Orange.

a) Find the production for the external demand $D = \begin{bmatrix} 2000 \\ 1500 \end{bmatrix}$
b) Which of the industries, if any, are profitable? Justify.
c) Is the economy productive? Justify.

(8) 6. Given $A = \begin{bmatrix} -2 & -3 & 4 \\ 0 & -1 & 3 \\ -3 & 8 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 130 \\ 65 \\ 0 \end{bmatrix}$

a) Find $adj(A)$
b) Find $\det(A)$
c) Find A^{-1}
d) Solve $AX = B$ using A^{-1}
e) Find ALL the solutions to the system $AX = \mathbf{0}$

(5) 7. Find the determinant of the matrix $A = \begin{bmatrix} 4 & -2 & 3 & 1 \\ -2 & -1 & 2 & 2 \\ 3 & 5 & 3 & -4 \\ -1 & 0 & -2 & 2 \end{bmatrix}$

(5) 8. Use Cramer's rule to solve the linear system: $\begin{matrix} 15x - 9y = 13 \\ 7x + 12y = 3 \end{matrix}$

(4) 9. Let A and B be 2×2 matrices such that $\det(A) = -4$ and $\det(B) = 11$. Find
a) $\det(3B)$ b) $\det(A^2 B^t)$ c) $\det[(BA)^{-1}]$

(5) 10. Let A(2,-2,3,3,-2), B(4,5,4,5,4), C(0,3,1,4,2) and D(1,5,1,3,6) be points in $\mathbf{R}^5$.
a) Find the vector $\overrightarrow{AB}$
b) Find the magnitude $\left|\overrightarrow{AB}\right|$
c) Find an equation of the line L that passes through the point C and that is parallel to vector $\overrightarrow{AB}$
d) Determine if the point D is on the line L

(5) 11. Consider the set of vectors S = { (s,t,st) | s,t ε **R** }.
a) Is **0** in S? Justify.
b) Is S closed under addition? Justify.
c) Is S closed under scalar multiplication? Justify.
d) Is S a subspace of $\mathbf{R}^3$? Justify.

(5) 12. Let $A = \begin{bmatrix} 3 & 8 & -5 \\ 1 & 4 & -1 \\ -2 & -6 & 3 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 6 \\ k \\ 2 \end{bmatrix}$

a) Find a relationship between x, y and z so that vector **u** is in Col(A)
b) Find k so that vector **v** is in Nul(A)

(5) 13. Suppose A is an 8x5 matrix
a) What is the maximum value for the rank of A?
b) What is the minimum value of Dim(Nul(A))?
c) What is Dim(Col(A)) if the rank of A is 4?
d) If A has 5 pivots (5 leading ones), what is Dim(Nul(A^T))?

(7) 14. Consider the matrix $A = \begin{bmatrix} -2 & -4 & -3 & 0 & 5 & 16 \\ 1 & 2 & 2 & -1 & -4 & -11 \\ -1 & -2 & -1 & -1 & 2 & 7 \\ 2 & 4 & 1 & 4 & 3 & 0 \end{bmatrix}$

which can be row reduced to $R = \begin{bmatrix} 1 & 2 & 0 & 3 & 0 & -3 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Let $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}, \mathbf{a_4}, \mathbf{a_5}, \mathbf{a_6}$ represent the column vectors of A.

a) Determine if the following sets of vectors are linearly dependent or independent. Briefly justify your answer.
i) $\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$ ii) $\{\mathbf{a_1}, \mathbf{a_3}, \mathbf{a_5}\}$ iii) $\{\mathbf{a_1}, \mathbf{a_4}, \mathbf{a_6}\}$
b) Find a basis for Col(A), and write the columns of A as a combination of these vectors. What is the dimension of Col(A)?
c) Find a basis for Nul(A), and give its dimension.

(6) 15. Solve the following linear program. Your solution should include both the maximum value of z and the corresponding basic feasible solution.

Maximize $z = 8x_1 - x_2 - 2x_3$
Subject to
$x_1 + 3x_2 - x_3 \le 3$
$3x_1 + 3x_2 \le 10$
$5x_1 + 10x_2 - 2x_3 \le 20$
$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$

(6) 16. Given the following linear program

Minimize	$z = -5x_1 + x_2$
Subject to	$-2x_1 + x_2 \le 2$
	$x_1 - 3x_2 \le 3$
	$x_1 \ge 0,\ x_2 \ge 0$

a) Use the simplex algorithm to show that z has no lower bound over the given feasibility region.
b) Find a specific solution with z = -435.

(7) 17. Josie needs a daily dose of at least 30 milligrams of iron and at least 100 milligrams of calcium. Suppose that one portion of spinach contains 15 milligrams of iron and 60 milligrams of calcium and that one portion of broccoli contains 10 milligrams of iron and 30 milligrams of calcium. Further suppose spinach costs 50 cents per portion and that broccoli costs 30 cents per portion. Finally, assume that Josie does not feel like eating more than 3 portions of the same vegetable per day. Find a daily diet such that the cost is a minimum (partial portions are permitted).

a) Define your variables using the expression "the number of ".
b) State the objective function in term of these variables and state whether it is to be maximized or minimized.
c) Set-up the constraints in terms of these variables.
d) Sketch the feasibility region and identify each of the corner points.
e) Use the graphical method to solve the linear program.

Answers

1. a) inconsistent b) $\left(-2t+1, \frac{3}{11}t - \frac{7}{11}, -\frac{4}{11}t + \frac{2}{11}, t\right)$

2. a) k = -1 b) k = 1 c) $k \neq -1,1$

3. a) x = number of Apple Cakes sold, y = number of Chocolate Mousse Cakes sold, z = number of Strawberry Cheesecakes sold

$$\begin{cases} x+y+z=20 \\ 12x+14y+16z=300 \end{cases}$$

b) (t – 10,30 – 2t,t) c) (0,10,10) and (1,8,11)

4. a) $\begin{bmatrix} -\frac{1}{19} & \frac{7}{38} \\ \frac{9}{19} & \frac{16}{19} \end{bmatrix}$ b) not possible c) $\begin{bmatrix} 36 & 23 \\ -22 & 25 \\ 24 & 9 \end{bmatrix}$ d) $\begin{bmatrix} -3 & -22 & 4 \\ -15 & 5 & -12 \end{bmatrix}$

5. a) (6000,7000) b) Yellow is profitable. Its column sum is less than 1.

c) Economy is productive. Row sums are both less than 1.

6. a) $\begin{bmatrix} -25 & 35 & -5 \\ -9 & 10 & 6 \\ -3 & 25 & 2 \end{bmatrix}$ b) 65 c) $\frac{1}{65}\begin{bmatrix} -25 & 35 & -5 \\ -9 & 10 & 6 \\ -3 & 25 & 2 \end{bmatrix}$ d) $\begin{bmatrix} -15 \\ -8 \\ 19 \end{bmatrix}$ e) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

7. -215

8. $x = \frac{61}{81}$ and $y = -\frac{46}{243}$

9. a) 99 b) 176 c) $-\frac{1}{44}$

10. a) (2,7,1,2,6) b) $\sqrt{94}$ c) **x** = (0,3,1,4,2) +t(2,7,1,2,6) d) No

11. a) Yes b) No c) No d) No

12. a) x + y +2z = 0 b) k = -1

13. a) 5 b) 0 c) 4 d) 3

14. a)i) LD ii)LI iii) LI b) $\{\mathbf{a}_1,\mathbf{a}_3,\mathbf{a}_5\}$, $\mathbf{a}_2 = 2\mathbf{a}_1$, $\mathbf{a}_4 = 3\mathbf{a}_1 - 2\mathbf{a}_3$, $\mathbf{a}_6 = -3\mathbf{a}_1 + 2\mathbf{a}_5$, Dim(Col(A)) = 3
c) {(-2,1,0,0,0,0), (-3,0,2,1,0,0), (3,0,0,0,-2,1)}, Dim(Nul(A)) = 3

15. max(z) = 26, $\left(\frac{10}{3},0,\frac{1}{3},0,0,4\right)$

16. b) (93,30,158,0)

17. a) x = the number of portions of spinach, y = the number of portions of broccoli
b) minimize z = 50x + 30y c) $15x + 10y \geq 30$
$60x + 30y \geq 100$
$0 \leq x \leq 3,\ 0 \leq y \leq 3$
d) (2,0), (3,0), (3,3), (1/6, 3), (2/3, 2)
e) The minimum cost is 93.33 cents and occurs when Josie eats 2/3 of a portion of spinach and 2 portions of broccoli.