(Marks)

(12) 1. Use algebraic techniques to evaluate the following limits. Identify the limits that do not exist and use $-\infty$ or ∞ as appropriate. Show your work.

(a) $\lim_{x\to -3} \dfrac{x^2-2x-15}{x+3}$

(b) $\lim_{x\to -2} \dfrac{\sqrt{x+3}-1}{x+2}$

(c) $\lim_{x\to -\infty} \dfrac{2x-6}{x+1}$

(d) $\lim_{x\to \infty} \left(3+\dfrac{5}{\sqrt{x}}\right)$

(e) $\lim_{x\to 2^-} f(x)$, where $f(x) = \begin{cases} 3x-1 & \text{if } x<2 \\ x^2+4 & \text{if } x \geq 2 \end{cases}$

(f) $\lim_{x\to -3^+} \dfrac{x+2}{x^2+6x+9}$

(4) 2. Use the graph of the function $f(x)$ below to find the following. Use ∞, $-\infty$, or DNE where appropriate.

(a) $\lim_{x\to -\infty} f(x)$ = ____________

(b) $\lim_{x\to -3^-} f(x)$ = ____________

(c) $\lim_{x\to -3^+} f(x)$ = ____________

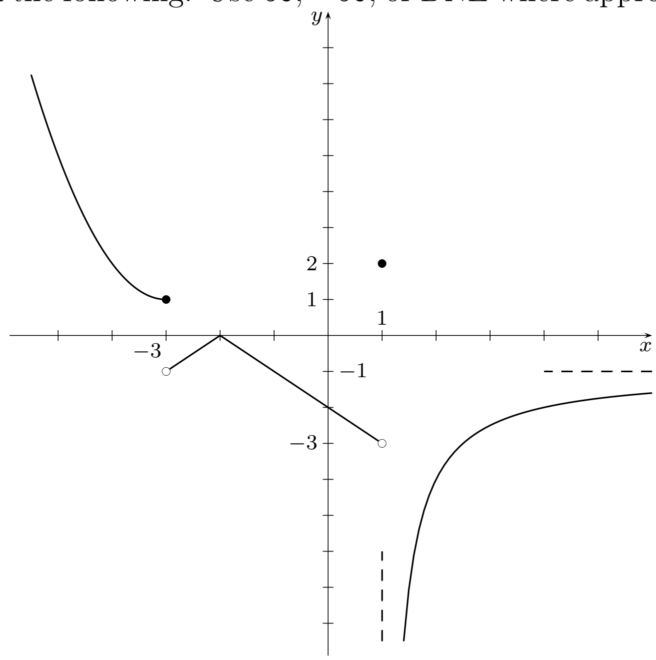
(d) $\lim_{x\to 1^-} f(x)$ = ____________

(e) $\lim_{x\to 1^+} f(x)$ = ____________

(f) $\lim_{x\to \infty} f(x)$ = ____________

(g) $f(-2)$ = ____________

(h) $f(1)$ = ____________



(3) 3. For the function $f(x)$ as defined in question number 2,

(a) list the value(s) of x where $f(x)$ is discontinuous,

(b) list the value(s) of x where $f(x)$ is continuous but not differentiable.

(2) 4. Find the value(s) of x for which the following function is not continuous. Justify using the definition of continuity.

$$f(x) = \begin{cases} 24-x^2 & \text{if } x \leq 5 \\ |4-x| & \text{if } x > 5 \end{cases}$$

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(2) 5. Find the value(s) of k that will make $f(x)$ continuous for all real numbers.

$$f(x) = \begin{cases} \dfrac{x^2+2x-3}{x-1} & \text{if } x \neq 1 \\ \\ k^2 & \text{if } x = 1 \end{cases}$$

(8) 6. Given $f(x) = \sqrt{5-x}$,

(a) find $f'(x)$ **using the limit definition of the derivative**,

(b) find the equation of the tangent line to $f(x)$ at $x = 1$.

(24) 7. Find $\dfrac{dy}{dx}$ for each of the following functions. **Do not simplify your answers.**

(a) $y = 5x\sqrt{x^2+1}$

(b) $y = \dfrac{6x^2}{x^{1/2}} + \dfrac{5}{2x} - \dfrac{e}{\sqrt{x}}$

(c) $y = \dfrac{\sin(3x)}{3 - 2\cos x}$

(d) $y = \tan(2x) + e^{\sec 3x}$

(e) $y = \ln\left(\dfrac{\cos x}{\sqrt[3]{x^3+2}\,(3x-1)^2}\right)$

(f) $y = \log_7(x^4 - 3x) + 2^{3x}$

(g) $e^y = x^2y^3 + 4$

(h) $y = (3x^2+5)^{\sqrt{x}}$

(3) 8. Given the function $f(x) = x\sin(2x)$, find $f''(0)$.

(4) 9. Find the x-values of the points on the graph of $f(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 - x$ where the slope of the tangent line is 2.

(4) 10. Find the absolute extrema of $f(x) = 2x^3 - 9x^2 + 3$ on the interval $[2, 5]$.

(10) 11. Given $f(x) = \dfrac{2x^2}{(x-1)^2}$ $\quad f'(x) = \dfrac{-4x}{(x-1)^3}$ $\quad f''(x) = \dfrac{4(2x+1)}{(x-1)^4}$

(a) find all x and y intercepts, vertical and horizontal asymptotes, intervals where $f(x)$ is increasing or decreasing, relative extrema, intervals where $f(x)$ is concave up or concave down, and points of inflection.

(b) sketch the graph of $f(x)$ on the following page.

(4) 12. Given $f(x) = 3x^4 - 18x^2 + 24x + 5$ $\quad f'(x) = 12(x-1)^2(x+2)$ $\quad f''(x) = 36x^2 - 36$
find all relative extrema of $f(x)$.

(5) 13. A car dealer sells 80 cars per month at a price of \$20 000 per car. For every decrease of \$1000 in the price, 10 more cars are sold. What is the price to maximize the revenue?
(Be sure to use a test to confirm that this is a maximum.)

(4) 14. The average cost function in \$/unit is given by $\overline{C} = -2x^2 + 3x + \frac{10}{x}$ where x is the number of units produced. Find the marginal cost at $x = 50$ and interpret the result.

(Marks)

(5) 15. A company wants to build a rectangular fence next to the wall of a building. No fencing is required along the wall of the building. The fence costs \$40 per meter. The company has \$4000 to spend for the fence. Find the dimensions of the courtyard that will maximize the area.
(Use a test to confirm that this is a maximum.)

(6) 16. Given the demand function $p = \dfrac{2}{\sqrt{7+x^2}}$,

(a) find the price elasticity of demand function (simplify your answer),

(b) find the price elasticity of demand at $x = 3$,

(c) determine whether the demand is elastic or inelastic at $x = 3$. (Justify your answer.)

Answers

(1a) -8 ; (1b) $\frac{1}{2}$; (1c) 2 ; (1d) 3 ; (1e) 5 ; (1f) $-\infty$

(2a) $+\infty$; (2b) 1 ; (2c) -1 ; (2d) -3 ; (2e) $-\infty$; (2f) -1 ; (2g) 0 ; (2h) 2

(3a) $x = -3$, $x = 1$; (3b) $x = -2$; (4) discontinuous at $x = 5$; (5) $k = \pm 2$

(6a) Use $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to find $f'(x) = \dfrac{-1}{2\sqrt{5-x}}$; (6b) $y = -\frac{1}{4}x + \frac{9}{4}$

(7a) $\dfrac{dy}{dx} = 5\sqrt{x^2+1} + 5x \dfrac{2x}{2\sqrt{x^2+1}}$; (7b) $\dfrac{dy}{dx} = 9x^{1/2} - \frac{5}{2}x^{-2} + \frac{e}{2}x^{-3/2}$

(7c) $\dfrac{dy}{dx} = \dfrac{3\cos 3x(3 - 2\cos x) - 2\sin x \,.\, \sin 3x}{(3 - 2\cos x)^2}$; (7d) $\dfrac{dy}{dx} = 2\sec^2(2x) + e^{\sec 3x} \,.\, 3\sec 3x \tan 3x$

(7e) $\dfrac{dy}{dx} = \dfrac{-\sin x}{\cos x} - \dfrac{1}{3}\dfrac{3x^2}{x^3+2} - 2\dfrac{3}{3x-1}$; (7f) $\dfrac{dy}{dx} = \dfrac{4x^3 - 3}{(x^4 - 3x)\ln 7} + 3\,(2^{3x})\,\ln 2$

(7g) $\dfrac{dy}{dx} = \dfrac{2xy^3}{e^y - 3x^2y^2}$; (7h) $\dfrac{dy}{dx} = (3x^2+5)^{\sqrt{x}} \left[\dfrac{1}{2\sqrt{x}} \ln(3x^2+5) + \sqrt{x}\,\dfrac{6x}{3x^2+5}\right]$

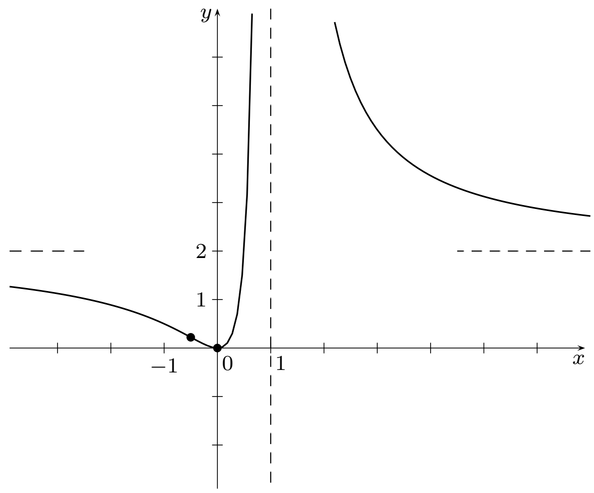
(8) $f''(0) = 4$; (9) $x = -1$, $x = \frac{3}{2}$

(10) absolute maximum is 28 at $x = 5$; absolute minimum is -24 at $x = 3$

(Marks)

(11)

x and y intercept: (0,0)
vertical asymptote: $x = 1$
horizontal asymptote: $y = 2$
relative minimum: $(0, 0)$
points of inflection: $\left(-\frac{1}{2}, \frac{2}{9}\right)$
increasing: $0 < x < 1$
decreasing: $x < 0$ or $x > 1$
concave up: $-\frac{1}{2} < x < 1$ or $x > 1$
concave down: $x < -\frac{1}{2}$



(12) relative minimum at $(-2, -67)$; (13) the price is \$14,000 to maximize the revenue

(14) the marginal cost is $C'(50) = -14\,700$ dollars/unit ; $C'(50) \approx C(51) - C(50)$

(15) the dimensions are 25 by 50 meters to maximize the area.

(16a) $\eta = -\dfrac{x^2 + 7}{x^2}$; (16b) $\eta(3) = -\dfrac{16}{9} \approx -1.78$; (16c) demand is elastic