(Marks)

1. Evaluate the derivative $\dfrac{\mathrm{d}}{\mathrm{d}x}\sin\left(\arccos\left(\sqrt{1-x^2}\right)\right)$ and *simplify your answer.*

2. Evaluate the following integrals.

   (a) $\displaystyle\int_1^{\sqrt{2}} \frac{4+2\sqrt{x^2-1}}{x\sqrt{x^2-1}}\,\mathrm{d}x$ (b) $\displaystyle\int_1^5 \frac{x+2}{\sqrt{2x-1}}\,\mathrm{d}x$ (c) $\displaystyle\int \mathrm{e}^{-2x}\cos(6x)\,\mathrm{d}x$

   (d) $\displaystyle\int \sqrt{t+1}\ln\sqrt{t+1}\,\mathrm{d}t$ (e) $\displaystyle\int_0^{\pi/2} \sin^3 x\cos^3 x\,\mathrm{d}x$ (f) $\displaystyle\int \frac{\mathrm{d}x}{x^2\sqrt{x^2-36}}$

   (g) $\displaystyle\int \frac{x+4}{x(x^2+2)}\,\mathrm{d}x$

3. Evaluate the following improper integrals.

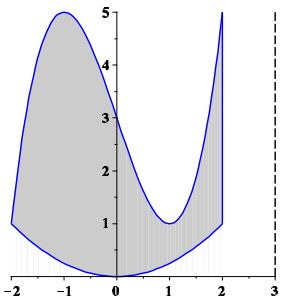
   (a) $\displaystyle\int_2^{\infty} \frac{1}{1-x^2}\,\mathrm{d}x$ (b) $\displaystyle\int_0^2 \frac{x}{x^2-4}\,\mathrm{d}x$

4. Evaluate the following limits.

   (a) $\displaystyle\lim_{x\to 0^+} \frac{(\ln x)^2}{1+x^{-1}}$ (b) $\displaystyle\lim_{x\to 0}(\sec x)^{\cot^2 x}$ (c) $\displaystyle\lim_{x\to\infty}\left(\frac{x^2+2}{x-3}-\frac{(x-2)^3}{x^2+1}\right)$

5. Find the area of the region (in quadrant I) bounded by $y=\dfrac{2}{x}$, $y=\dfrac{3x}{x^2+2}$ and $x=1$.
   *Give the exact answer in simplified form only: no decimals.*

6. Let $\mathcal{R}$ be the region bounded by the graphs $y=\dfrac{x^2}{4}$, $y=x^3-3x+3$, $x=-2$ and $x=2$.

   (a) Set up, *but do not evaluate*, the integral required to find the volume of the solid generated by revolving $\mathcal{R}$ about the $x$-axis.

   (b) Find the volume of the solid generated by revolving $\mathcal{R}$ about the line $x=3$.
   *Give the exact answer in simplified form only: no decimals.*

7. Find a solution to the differential equation $y'=\dfrac{\sqrt{1-y^2}}{1+x^2}$ that satisfies the initial condition $y(1)=0$.

8. Let $\displaystyle\sum_{n=1}^{\infty} a_n$ be a series whose n$^{\text{th}}$ partial sum is given by $s_n=\dfrac{2n+1}{n+2}$.

   (a) Evaluate $\displaystyle\sum_{n=1}^{\infty} a_n$. (b) Find $a_5$.

9. Based only on the value of $\displaystyle\lim_{n\to\infty} a_n$ what can you say about the convergence of $\displaystyle\sum_{n=1}^{\infty} a_n$ for each of the following?

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{\cos n}{n}$ (b) $\dfrac{1}{2}+1+\dfrac{1}{4}+1+\dfrac{1}{8}+1+\dfrac{1}{16}+1+\cdots$

(Marks)

10. Determine whether each of the following series converges or diverges; if it converges, find the sum. *Justify your answers.*

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{5(-4)^{n+2}}{3^{2n+1}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{2n-1}{2n+1}\right)$

11. Determine whether each of the following series converges or diverges. *State the tests you use and verify that the conditions for using them are satisfied.*

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ (b) $\displaystyle\sum_{k=1}^{\infty} \frac{\cos^2 k}{k\sqrt{k}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\mathrm{e}^{\sqrt{n}}}{\sqrt{n}}$ (d) $\displaystyle\sum_{n=2}^{\infty} \sin\left(\frac{2}{n}\right)$

12. Determine whether each of the following series converges absolutely, conditionally or diverges. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{-n}{2n+1}\right)^{3n}$ (b) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$

13. Find the radius and interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n (x-2)^{n+1}}{2n+1}$.

14. Find the Taylor series of $f(x) = \cos 2x$ centred at $\pi/2$. State the first four *non-zero* terms and give the formula for the $\mathrm{n}^{\mathrm{th}}$ term.

ANSWERS

1. $\frac{x}{|x|}$
2. (a) $\pi + \ln 2$ (b) $\frac{28}{3}$ (c) $\frac{\mathrm{e}^{-2x}}{20}(3\sin 6x - \cos 6x) + C$ (d) $(t+1)^{2/3}\left(\frac{1}{3}\ln(t+1) - \frac{2}{9}\right) + C$
(e) $\frac{1}{12}$ (f) $\frac{\sqrt{x^2-36}}{36x} + C$ (g) $2\ln|x| - \ln(x^2+2) + \frac{1}{\sqrt{2}}\arctan\frac{x}{\sqrt{2}} + C$
3. (a) $-\frac{1}{2}\ln 3$ (b) DIV (to $-\infty$)
4. (a) 0 (b) $\sqrt{\mathrm{e}}$ (c) 9
5. $\displaystyle\int_1^2 \left(\frac{2}{x} - \frac{3x}{x^2+2}\right) \mathrm{d}x = \ln\sqrt{2}$
6. (a) $\pi\displaystyle\int_{-2}^{2} \left((x^3-3x+3)^2 - (x^2/4)^2\right) \mathrm{d}x$ (b) $2\pi\displaystyle\int_{-2}^{2} (3-x)\left((x^3-3x+3) - (x^2/4)\right) \mathrm{d}x = \frac{352\pi}{5}$
7. $\arcsin y = \arctan x - \frac{\pi}{4}$ **or** $y = \sin\left(\arctan x - \frac{\pi}{4}\right)$ $x \geq -1$
8. (a) 2 (b) $\frac{1}{14}$
9. (a) $\lim a_n = 0 \Rightarrow$ no conclusion. (b) $\lim a_n \neq 0 \Rightarrow \sum a_n$ DIV
10. (a) $-\frac{320}{39}$ (b) DIV (to $-\infty$)
11. (a) CONV (RT) (b) CONV (DCT with $\sum \frac{1}{k^{3/2}}$) (c) DIV (NTT) (d) DIV (LCT with $\sum \frac{1}{n}$)
12. (a) CONV ABS (NRT) (b) CONV COND (DCT with $\sum \frac{1}{n^{1/2}}$ & AST)
13. $\frac{1}{3}$, $\left[\frac{5}{3}, \frac{7}{3}\right)$
14. $-1 + 2(x-\frac{\pi}{2})^2 - \frac{2}{3}(x-\frac{\pi}{2})^4 + \frac{4}{45}(x-\frac{\pi}{2})^6 - \cdots = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1} 2^{2n} (x-\frac{\pi}{2})^{2n}}{(2n)!}$