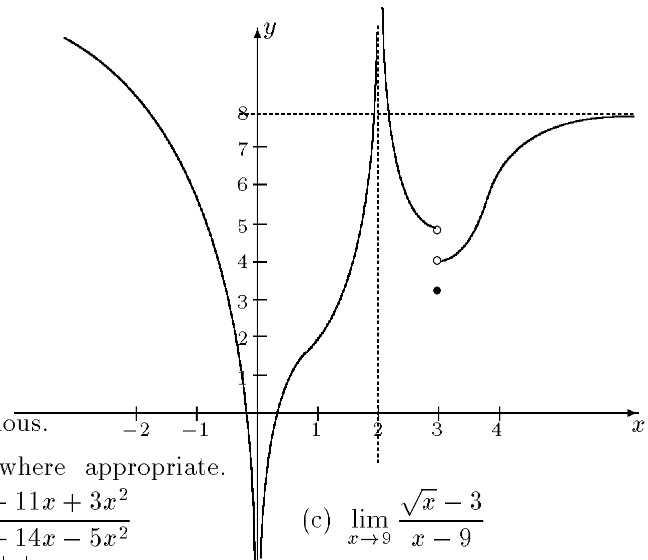
1. Refer to the sketch below to evaluate the following. If a value does not exist, state in which way ($+\infty$, $-\infty$, or "does not exist").
   (a) $\lim_{x\to-\infty} f(x) =$
   (b) $\lim_{x\to 0} f(x) =$
   (c) $\lim_{x\to 2} f(x) =$
   (d) $\lim_{x\to 3^-} f(x) =$
   (e) $\lim_{x\to 3^+} f(x) =$
   (f) $f(3) =$
   (g) $\lim_{x\to+\infty} f(x) =$
   (h) List the values of x where $f(x)$ is discontinuous.



2. Evaluate the limits. Use the symbols $-\infty$ or $+\infty$ where appropriate.
   (a) $\lim_{x\to+\infty} \dfrac{6-11x+3x^2}{3+14x-5x^2}$ (b) $\lim_{x\to 3} \dfrac{6-11x+3x^2}{3+14x-5x^2}$ (c) $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$
   (d) $\lim_{x\to 0} \dfrac{\sin 2x}{\tan x}$ (e) $\lim_{x\to 0^+} \dfrac{3|x|-x}{x^2}$

3. Find all the vertical and horizontal asymptotes for $f(x) = \dfrac{3x+\sqrt{x^2+1}}{2x-1}$.

4. (a) State the definition for a function $f(x)$ to be continuous at $x = a$.
   (b) Using this definition, determine if the following function $f(x)$ is continuous at $x = 0$. (Show your justification for your answer.)
   $$f(x) = \begin{cases} (x-1)^2 & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ \cos(x) & \text{if } x > 0 \end{cases}$$
   (c) Sketch the graph of the function $y = f(x)$ in part (b).

5. Sketch (if possible) a function f that is continuous but not differentiable at $x = 0$.

6. State a limit definition of the derivative for a function $f(x)$.
   Use the above definition to find the derivative of $f(x) = \dfrac{2}{1-x}$.

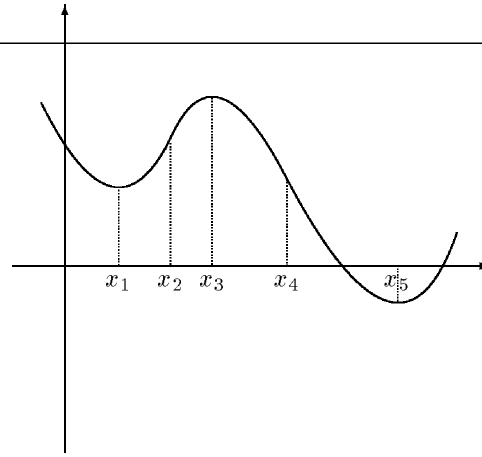
7. For each of the following functions, calculate the derivative $\dfrac{dy}{dx}$. You do **NOT** have to simplify your answers.
   (a) $y = \dfrac{x^7}{7} - \dfrac{2}{\sqrt[5]{x^2}} + \dfrac{\sqrt{x}}{3} - \ln 2 + 3^x$ (b) $y = x^8 \sec x$
   (c) $y = \dfrac{\tan^3(x+1)}{\ln(3x-2)}$ (d) $y = \sqrt{\sin(e^x + e^2)}$
   (e) $y = (x+1)^{(x^2+1)}$

8. Find the equation of the tangent line to the graph of $y = \dfrac{\cos(x-1)}{x+1}$ at the point where $x = 1$.

9. Given $x^3 + y^3 = 6xy + 1$:
   (a) find $\dfrac{dy}{dx}$ using implicit differentiation;
   (b) evaluate $\dfrac{dy}{dx}$ at $(1, 0)$.

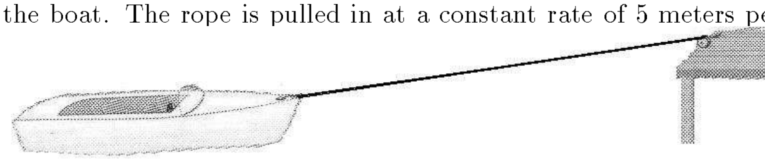
10. Find the absolute maximum and minimum values of $h(x) = \dfrac{x^2-1}{x^2+1}$ on the interval $[-1, 1]$.

11. The graph of $y = f(x)$ is given at right.



   (a) At what value(s) of x does $f'(x)$ change sign?
   (b) At what value(s) of x does $f'(x)$ have a local (relative) maximum?
   (c) At what value(s) of x does $f'(x)$ have a local (relative) minimum?
   (d) Sketch a rough graph of $f'(x)$ (on the same axes).
   (e) At what value(s) of x does $f''(x)$ change sign?

12. A boat is pulled into a dock by a rope attached to the bow of the boat, passing through a pulley on the dock that is 10 meters higher than the bow of the boat. The rope is pulled in at a constant rate of 5 meters per minute.



   (a) If s is the length (in meters) of rope between the pulley and the bow of the boat, express s as a function of the angle θ between the water and the rope.
   (b) At what rate is the angle θ changing when the length $s = 26$ meters?

13. For the function f, given below with its derivatives:

$$f(x) = \frac{x-2}{x^3} \qquad f'(x) = \frac{6-2x}{x^4} \qquad f''(x) = \frac{6x-24}{x^5}$$

sketch the graph, identifying all intercepts, local extrema, and inflection points. Specify intervals where the graph is increasing, decreasing, concave up, and concave down. Show all your work.

14. A rectangular box has an open top, a square base, and a surface area of 147 square meters; find the dimensions of the box if it is to have the maximum possible volume.

15. Given that $f'(x) = x + \sin(x)$, and $f(0) = 3$, find $f(x)$.

16. Evaluate the following integrals:
   (a) $\displaystyle\int \frac{x^4 - 4x - 1}{2x^2}\, dx$
   (b) $\displaystyle\int \left(\frac{4}{t^5} - \frac{t^5}{4} + 5\,e^t - \frac{1}{e^5}\right) dt$
   (c) $\displaystyle\int \sec x(\tan x - \sec x)\, dx$
   (d) $\displaystyle\int_1^4 (\sqrt{x} + 2)^2\, dx$

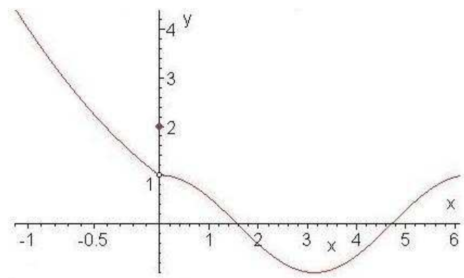
17. (a) Evaluate the Riemann sum for $f(x) = \sin(x)$, $0 \le x \le \pi$, with three equal subintervals, taking the sample points to be the midpoints.
   (b) Evaluate $\displaystyle\int_o^{\pi} \sin\theta\, d\theta$.

18. Suppose $F(x) = \displaystyle\int_0^{\sqrt{x}} e^{t^2}\, dt$.
   (a) What is $F(0)$?
   (b) What is $F'(x)$? *(Hint: you may wish to use the Fundamental Theorem of Calculus.)*

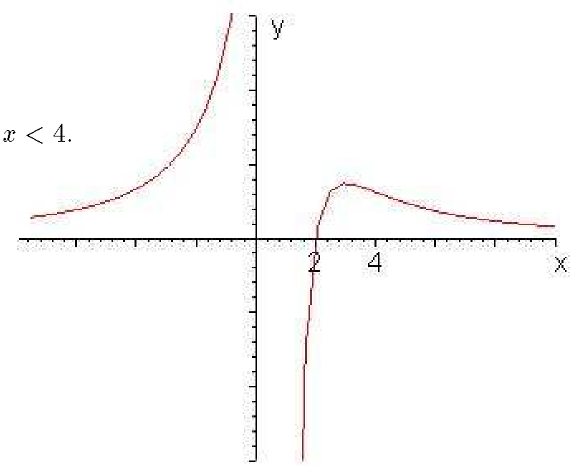
19. Find the area of the region bounded by the curve $y = x^2 - 2x$ and the x-axis.

Answers

1. (a) $+\infty$ (b) $-\infty$ (c) $+\infty$ (d) 5 (e) 4 (f) 3 (g) 8 (h) $0, 2, 3$
2. (a) $-\frac{3}{5}$ (b) $-\frac{7}{16}$ (c) $\frac{1}{6}$ (d) 2 (e) $+\infty$
3. VA: $x = \frac{1}{2}$ HA: $y = 1$ and $y = 2$
4. (a) (i) $f(a)$ exists (ii) $\lim_{x\to a} f(x)$ exists (iii) $f(a) = \lim_{x\to a} f(x)$.
   (b) Discontinuous at $x = 0$ because $f(0) = 2 \neq 1 = \lim_{x\to 0} f(x)$ (c) Graph:



5. Many examples, such as the "V-shaped" absolute value graph.
6. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$ (if the limit exists)
   $$f'(x) = \lim_{h\to 0} \frac{\frac{2}{1-(x+h)} - \frac{2}{1-x}}{h} = \lim_{h\to 0} \frac{(2-2x)-)2-(2x+2h))}{h(1-x-h)(1-x)} = \lim_{h\to 0} \frac{2}{(1-x-h)(1-x)} = \frac{2}{(1-x)^2}$$
7. (a) $x^6 + \frac{4}{5}x^{-7/5} + \frac{1}{6}x^{-1/2} + 3^x \ln 3$
   (b) $x^8 \sec x \tan x + 8x^7 \sec x$
   (c) $\dfrac{3\tan^2(x+1)\sec^2(x+1)\ln(3x-2) - 3\tan^3(x+1)/(3x-2)}{\ln^2(3x-2)}$
   (d) $\frac{1}{2}(\sin(e^x + e^2))^{-1/2}\cos(e^x + e^2)\cdot e^x$
   (e) $(x+1)^{x^2+1}\left(\frac{x^2+1}{x+1} + 2x\ln(x+1)\right)$
8. $y = -\frac{1}{4}x + \frac{3}{4}$
9. (a) $\dfrac{2y - x^2}{y^2 - 2x}$ (b) $\frac{1}{2}$
10. Abs max at $(-1, 0), (1, 0)$ Abs min at $(0, -1)$
11. (a) x_1, x_3, x_5 (b) x_2 (c) x_4 (d) /\/ zigzag shape, crossing the x axis at x_1, x_3, x_5.
    (e) x_2, x_4.
12. (a) $s = 10\csc\theta$ (b) $25/312$
13. VA at $x = 0$, HA at $y = 0$, x-intercept at $x = 2$,
    CP (local max) at $x = 3$, PI at $x = 4$,
    increasing for $x < 0$ and $0 < x < 3$, and decreasing for $x > 3$,
    concave up for $x < 0$ and for $x > 4$, and concave down for $0 < x < 4$.
    Graph at right.



14. $7\text{m} \times 7\text{m} \times \frac{7}{2}\text{m}$
15. $f(x) = \frac{1}{2}x^2 - \cos x + 4$
16. (a) $\frac{1}{6}x^3 - 2\ln|x| + \frac{1}{2x} + C$ (b) $-t^{-4} - \frac{1}{24}t^6 + 5e^t - \frac{1}{e^5}t + C$
    (c) $\sec x - \tan x + C$ (d) $\frac{229}{6}$
17. (a) $\frac{2\pi}{3}$ (b) 2
18. (a) 0 (b) $\frac{e^x}{2\sqrt{x}}$
19. $\frac{4}{3}$ [**not** $-\frac{4}{3}$!]