

Mathematical Models 2

201-225 Instructor: Bob DeJean

Winter 2009

Final Exam

Answers to 4 decimal places please.

2 mark question

What is the Root Mean Square of the function $y = 20 \sin(60\pi t)$?

Is $y = 3x^2 - 5$ a solution of $xy' - 2y = 10$?

3 mark questions

Find the derivatives

$$y = 12 \tan^{-1}(2x+3)$$

$$y = x^3 \ln(x - 2)$$

$$y = 17^{3x-5}$$

$$y = \frac{e^{2x} - 1}{e^{2x} + 1}$$

Integrate

$$\int 12x^3 - \frac{10}{x^6} dx =$$

$$\int_1^2 (4x - 3)^2 dx =$$

$$\int_4^9 3 + \sqrt{x} dx =$$

$$\int_0^{\pi/2} 15 \sin^4 3x \cos 3x dx =$$

$$\int \frac{5e^x}{3e^x - 1} dx =$$

$$\int x^2 4^{x^3} dx =$$

$$\int \frac{dx}{\sec 5x} =$$

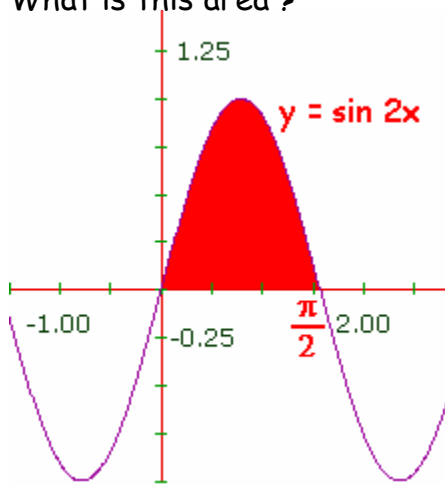
4 mark questions

$$\int \sin^2 x \cos^3 x \, dx =$$

$$\int \frac{dx}{\sqrt{16-x^2}} =$$

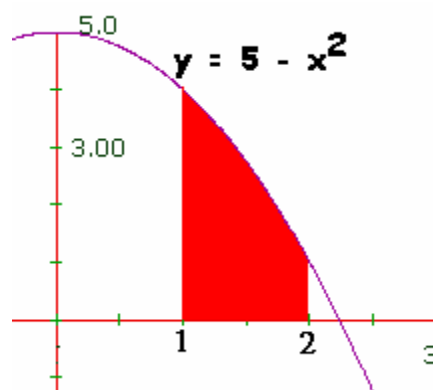
$$\int \frac{10}{x^2 + 6x + 10} \, dx =$$

What is this area ?



Find the area under the curve $y = 4x^3 + x - 1$ between $x = 2$ and $x = 3$.

Here's an area:
Approximate the area
using the Trapezoidal
Rule with $n = 5$



Solve for y $xy' + 3y^2 = 0$

Find y: $y' - y^2 = 16$ When $x = 0, y = 0$

Find y: $y' + \frac{y}{x} = 2$

What is the equation of the tangent line to $y = 6x^2 - 24x$ at the point where $x = 5$?

Solve $\ln(x + 3) - 2x = 0$ using Newton's Method. Your answer should be accurate to 3 decimal places. Show your work.

What is the value of $\frac{dy}{dx}$ for $x^3 + y^3 - 4xy^2 + 67 = 0$ at the point $(2, 5)$?

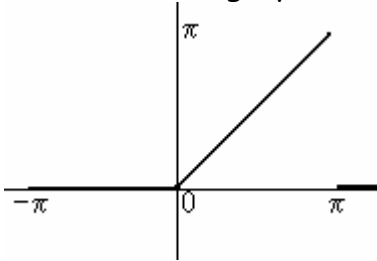
Your client wants you to design him a rectangular factory building. To save money, the sides and the back will be stucco. But to impress the clients, the front will be fieldstone and plate glass. In fact the cost of a single meter of the front wall will be triple the cost of a meter of the side and back walls. The building will be one storey high and will have an area of 3200 square meters. What should the length of the front wall be to keep the cost to a minimum?

Find the area between $y = x^2 - x$ and $y = 6x - 10$.

7 mark question

Here is a function $f(x) = \begin{cases} 0 & \text{if } -\pi < x < 0 \\ x & \text{if } 0 < x < \pi \end{cases}$

and here is its graph:



I want the first few terms of its Fourier Series.

What is a_0 ?

What is a_1 ?

What is b_1 ?

Use these answers to write the first part of the Fourier Series for $f(x)$.

Answers

14.14

yes

$$\frac{24}{(2x+3)^2 + 1}$$

$$3x^2 \ln(x-2) + \frac{x^3}{x-2}$$

$$17^{3x-5} (\ln 17)^3$$

$$\frac{4e^{2x}}{(e^{2x} + 1)^2}$$

$$3x^4 + 2x^{-5} + C$$

10.3333

27.6667

-1

$$\frac{5}{3} \ln |e^x - 1| + C$$

$$\frac{1}{3} \frac{4^{x^3}}{\ln 4} + C$$

$$\frac{1}{5} \sin 5x + C$$

$$\frac{1}{3} (\sin x)^3 - \frac{1}{5} (\sin x)^5 + C$$

$$\sin^{-1}\left(\frac{x}{4}\right) + C$$

$$10 \tan^{-1}(x+3) + C$$

$$-\frac{x \cos 5x}{5} + \frac{\sin 5x}{25} + C$$

1

66.5

2.66

$$-\frac{1}{y} = C - 3 \ln|x| \quad \text{or} \quad y = \frac{1}{3 \ln x - C}$$

$$\frac{1}{4} \tan^{-1}\left(\frac{y}{4}\right) = x \quad \text{or} \quad y = 4 \tan(4x)$$

$$y = x + C/x$$

$$y = 36x - 150$$

$$x \approx 0.6494$$

-17.6

40 m

4.5

$$\frac{\pi}{4}$$

$$-\frac{2}{\pi}$$

1

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi} \cos x + 1 \sin x + \dots$$