(Marks)

(15) 1. Use algebraic techniques to evaluate the following limits. If a limit fails to exist, use one of the symbols $-\infty$ or ∞ as appropriate.

(a) $\displaystyle\lim_{x\to 10^+} \frac{x+5}{-x+10}$

(b) $\displaystyle\lim_{x\to -2} \frac{x^3+3x^2+2x}{x^2-x-6}$

(c) Find $\displaystyle\lim_{x\to +\infty} \frac{(3+7x)(1-2x)}{4x^4+1}$

(d) $\displaystyle\lim_{x\to 0} \sin x \left(\frac{\sin x}{x} - \cot x\right)$

(e) $\displaystyle\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2}$

(4) 2. Given the function f defined by $f(x) = \dfrac{x+5}{x^2+2x-15}$

(a) Find both the values of x where $f(x)$ is discontinuous

(b) Find the limit of $f(x)$ as x approaches each of the values found in part (a)

(3) 3. Find constants a such that the function is continuous for all real numbers

$$f(x) = \begin{cases} 12 & x \le -3 \\ ax+3 & -3 < x < 5 \\ -12 & x \ge 5 \end{cases}$$

4. Complete each part below

(1) (a) State the limit definition of the derivative of a function $f(x)$.

(4) (b) Use the limit definition of the derivative to find $f'(x)$ for $f(x) = \sqrt{8x+17}$

(28) 5. Find $\dfrac{dy}{dx}$ for each of the following functions. **Do not simplify your answer.**

(a) $y = \dfrac{2}{3x} + e^{\sin x} - \dfrac{1}{\sqrt[3]{x^2}} + \ln 2$

(b) $y = \sqrt[3]{\dfrac{3x+2}{5x^2-1}}$

(c) $y = 3(\sin x)^{2x}$

(d) $y = \log(x+1) + x^3\,3^x$

(e) $y = \ln\left[\dfrac{\sqrt{x^2+1}\,(2x+1)^3}{\sqrt[3]{3x^4-2}}\right]$

(Hint: Use the properties of logarithmic functions to simplify the problem first)

(f) $xy^2 = e^{xy} - 3e^x$

(g) $y = \dfrac{e^{3-x}\sqrt{x+1}}{\cos 2x}$

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(5) 6. Let $f(x) = x^3\,(3x+4)^2$
Find the x-coordinates, if any, at which the graph of $f(x)$ has a horizontal tangent.

(5) 7. Find the equation of the tangent line to the graph of $f(x) = \dfrac{2+\sqrt{x}}{5x+1}$ at point $\left(1, \frac{1}{2}\right)$.

(4) 8. Use the second derivative test to find the relative (local) extrema of $f(x) = \frac{1}{2}\,x^4 - 4x^2 + 5$

(4) 9. Find the absolute extrema of $f(x) = 2x^4 - 36x^2 + 20$ on the interval $[-4, -1]$.

(11) 10. Given the function $f(x) = x^5 - 5x^4$
List all x and y intercepts, vertical and horizontal asymptotes, relative extrema, points of inflection, intervals where $f(x)$ is increasing, decreasing, concave up and concave down.
Use all the above and sketch a carefully labelled graph of $f(x)$

(5) 11. Mary has 1800 m of fence which will be used to enclose 3 sides of a rectangular field. The fourth side has a river and no fence is needed. What dimensions will give her maximum area?

(5) 12. Suppose the average cost is $\overline{c} = 100 + 3x + 0.1x^2$ and the demand is $p = 30x - 0.9x^2$

(a) Find the Profit function

(b) Find the marginal profit

(c) Evaluate the marginal profit when $x = 3$. Interpret the result.

(6) 13. The demand function for a certain product is $p = \sqrt{16-x}$ where p is the price per unit of the product in dollars and x is the number of units of the product.

(a) State the domain of the function

(b) Find the price elasticity of demand, η

(c) State the intervals where the function is elastic, inelastic and of unit elasticity

(d) Find the price elasticity of demand when $x = 9$

(e) At $x = 9$, if the price increased by 4% what is the change in demand?

(Marks)

Answers

1. a) $-\infty$ b) $-\frac{2}{5}$ c) 0 d) -1 e) 4

2. a) -5, 3 b) $\lim_{x\to -5} f(x) = -\frac{1}{8}$ and $\lim_{x\to 3} f(x) =$ D.N.E. 3. $a = -3$

4. a) $f'(x) = \lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$ b) $f'(x) = \dfrac{4}{\sqrt{8x+17}}$

5. a) $\dfrac{dy}{dx} = -\frac{2}{3}x^{-2} + \cos x\, e^{\sin x} + \frac{2}{3}x^{-5/3}$ b) $\dfrac{dy}{dx} = \frac{1}{3}\left(\dfrac{3x+2}{5x^2-1}\right)^{-2/3} \dfrac{3(5x^2-1) - 10x(3x+2)}{(5x^2-1)^2}$

c) $\dfrac{dy}{dx} = 3\,(\sin x)^{2x}\left[\dfrac{2x\,\cos x}{\sin x} + 2\,\ln(\sin x)\right]$ d) $\dfrac{dy}{dx} = \dfrac{1}{(x+1)\,\ln(10)} + x^3\,3^x\,\ln(3) + 3x^2\,3^x\ln(3)$

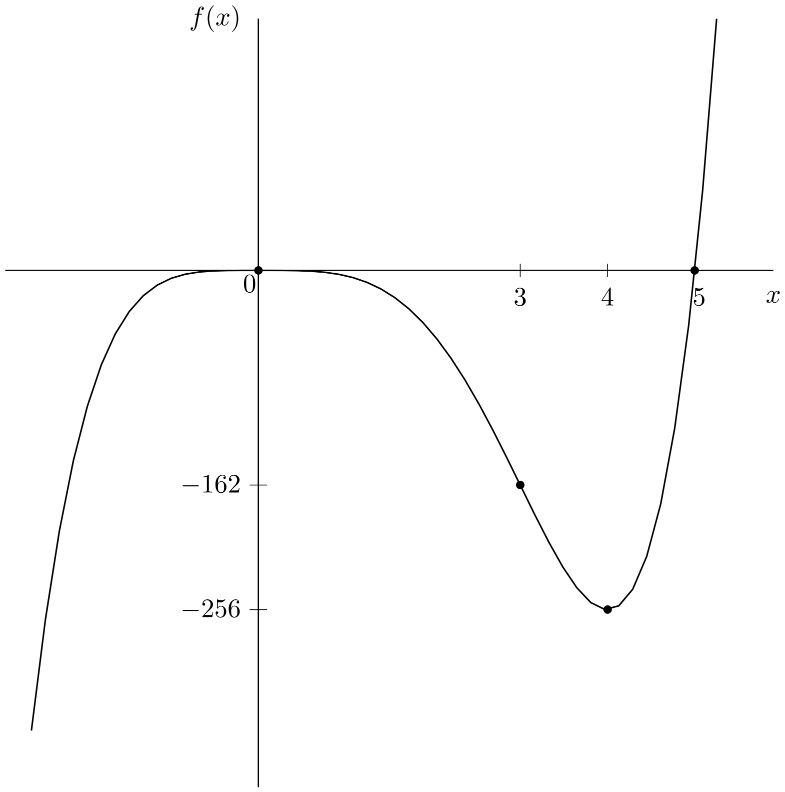
e) $\dfrac{dy}{dx} = \dfrac{x}{x^2+1} + \dfrac{6}{2x+1} - \dfrac{4x^3}{3x^4-2}$ f) $\dfrac{dy}{dx} = \dfrac{y\,e^{xy} - 3e^x - y^2}{2xy - x\,e^{xy}}$

g) $\dfrac{dy}{dx} = \dfrac{\left[-e^{3-x}\sqrt{x+1} + \frac{1}{2}(x+1)^{-1/2}\,e^{3-x}\right]\cos 2x - (-2\sin 2x)\;e^{3-x}\sqrt{x+1}}{\cos^2 2x}$

6. $x = -\frac{4}{3}$, $x = -\frac{4}{5}$, $x = 0$ 7. $y = -\frac{1}{3}x + \frac{5}{6}$ 8. Rel. Max:$(0,5)$, Rel. Min:$(-2,-3)$ and $(2,-3)$

9. absolute maximum is -14 at $x = -1$; absolute minimum is -142 at $x = -3$

10. x-int:(0,0), (5,0); y-int:(0,0); no asymptotes ; Rel. Max:(0,0); Rel. Min:$(4,-256)$; IP:$(3,-162)$; Dec:(0,4); Inc:$(-\infty,0)\cup(4,\infty)$; CD:$(-\infty,0)\cup(0,3)$; CU:$(3,\infty)$



11. Dim 450 m by 900 m

12. a) $P = -x^3 + 27x^2 - 100x$ b) $P' = -3x^2 + 54x - 100$

c) $P'(3) = 35$; $P'(3) \approx P(4) - P(3)$

13. a) $0 \le x \le 16$ b) $\eta = -\frac{32}{x} + 2 = \frac{2x-32}{x}$

c) elastic at $0 \le x < 10.67$; inelastic at $10.67 < x \le 16$; unit elasticity at $x = 10.67$

d) the demand will decrease by 6.24%