1 Various

1. Find all the real values of x for which the derivative of the function defined by
$k(x) = \dfrac{x^2}{e^x}$ is zero.

2. Find all the critical numbers for $y = \dfrac{e^{x^2}}{x^2}$

3. Find the point on the graph of $y = \sqrt{x}$ which is closest to the point $(4, 0)$.

4. Consider $f(x) = x^2 + 2x$ on $[0, 2]$. Find at least one value of x in the interval (0,2) for which the slope of the tangent to $y = f(x)$ is parallel to the line segment joining the points $(0, 0)$ and $(2, 8)$.

5. Find all critical numbers of the first derivative of $y = x^5 e^{-3x}$.

6. Given $y = x\sqrt{8 - x^2}$. Find all values of x such that $\dfrac{dy}{dx} = 0$.

7. If $y = (3x - 4)^(2x - 1)^2$ find all values of x for which $\dfrac{dy}{dx} = 0$.

8. Given $f(x) = \dfrac{x^2 + 2}{x^2 - 4}$
   (a) Find $f'(x)$ and simplify.
   (b) Find all critical values of f.
   (c) Find all vertical and horizontal asymptotes.
   (d) Find any absolute extrema on the interval $[-1, 2)$.

9. Find $f(x)$ given (i) $f(1) = e + 2$,
(ii) $f'(1) = e + 2$, and (iii) $f''(x) = e^x - \dfrac{1}{x^2}$.

10. Find all values of x such that $f'(x) = 0$ if $f(x) = (3 - x)^3(2x + 1)^2$

11. The position of a particle at time t is given by $s = \frac{1}{2}\sin 3t$. Find its velocity $\dfrac{ds}{dt}$, and its acceleration $\dfrac{d^2s}{dt^2}$ when $t = \frac{\pi}{6}$

12. Find f' and simplify. State all critical numbers for f.
   (a) $f(x) = x^4 \ln x$
   (b) $f(x) = \frac{x^{1/3}}{2x+1}$

13. Given the function $f(x) = \dfrac{x^2}{e^x}$, specify the interval(s) over which $f(x)$ is increasing.

14. Given $f(x) = 2\sin x + \sin 2x$, Determine if the function concave up, concave down, or neither when $x = \frac{\pi}{2}$. Give your reason.

15. Give the equations of all asymptotes of the function $y = \dfrac{x^2 + 15x - 16}{1 - x^2}$

Answers:

1. $x = 0, 2$
2. $x = 0, \pm 1$
3. $(\frac{7}{2}, \sqrt{\frac{7}{2}})$
4. 1.
5. 0, $5/3$
6. ± 2.
7. $1/2$ and $19/18$
8. (a) $f'(x) = \dfrac{-12x}{(x^2 - 4)^2}$
   (b) $x = 0$.
   (c) Vertical asymptotes: $x = \pm 2$
   Horizontal asymptote: $y = 1$
   (d) Absolute maximum at $(0, -1/2)$.
9. $f(x) = e^x + \ln|x| + x + 1$
10. $x = 3, -1/2, 9/10$
11. $\left.\dfrac{ds}{dt}\right|_{t=\pi/6} = 0$, $\left.\dfrac{d^2s}{dt^2}\right|_{t=\pi/6} = -9/2$
12. (a) $f'(x) = x^3(4\ln x + 1)$
    Critical numbers $x = 0, e^{-1/4}$
    (b) $\dfrac{1 - 4x}{3(2x + 1)^2 x^{2/3}}$
    Critical numbers $x = 1/4$
13. $[0, 2]$.
14. Concave down since $f''(\pi/2) = -2 < 0$.
15. Vertical asymptotes: $x = -1$ only
    Horizontal asymptote: $y = -1$