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Calculus II – Final Exam Problems  
**Positive Series**

Determine whether the series converge or diverge.

1.  $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+3)}$

2.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

3.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$

4.  $\sum_{n=1}^{\infty} \frac{n}{e^n}$

5.  $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{(n+1)^3}$

6.  $\sum_{n=1}^{\infty} \frac{2n+1}{n3^n}$

7.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt[6]{n^4}}$

8.  $\sum_{n=1}^{\infty} \frac{4}{n(n+1)(n+2)}$

9.  $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$

10.  $\sum_{n=1}^{\infty} \frac{(n+1)!}{(2n+1)!}$

11.  $\sum_{n=1}^{\infty} \frac{n^2+1}{n+5}$

12.  $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

13.  $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

14.  $\sum_{n=1}^{\infty} \frac{\pi}{(2n+3)(n-5)}$

15.  $\sum_{n=1}^{\infty} \frac{(n!)^2 2^n}{(2n)!}$

16.  $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+n}}{n^3}$

17.  $\sum_{n=2}^{\infty} \frac{n(1+n^2)}{n^3-3}$

18.  $\sum_{n=1}^{\infty} \frac{n!}{2^n 3^n}$

19.  $\sum_{n=1}^{\infty} \frac{1}{3+2^{-n}}$

20.  $\sum_{n=2}^{\infty} \frac{1}{n[\ln n]^{3/2}}$

21.  $\sum_{n=1}^{\infty} \frac{5^n+2^n}{4^n}$

22.  $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$

23.  $\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^8+6n^3}}$

24.  $\sum_{n=1}^{\infty} \ln \left( \frac{n+1}{n+2} \right)$

25.  $\sum_{n=2}^{\infty} \left( \frac{n}{\ln n} + \frac{3}{4^n} \right)$

26.  $\sum_{n=1}^{\infty} (e^{1/n} - 1)$

27.  $\sum_{n=1}^{\infty} \frac{\arctan n}{1+n^2}$

28.  $\sum_{n=1}^{\infty} \frac{n^3+1}{n!}$

29.  $\sum_{n=1}^{\infty} \frac{5n}{(n^7+3)^{1/3}}$

30.  $\sum_{n=1}^{\infty} \frac{n(n-1)}{(n+1)(n+2)}$

31.  $\sum_{n=2}^{\infty} \left( \frac{n}{\ln n} - \frac{5}{2^n} \right)$

32.  $\sum_{n=1}^{\infty} \frac{5n-6}{3n^{5/2}+2n-2}$

33.  $\sum_{n=1}^{\infty} \frac{n!}{2^n n^2}$

34.  $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

35.  $\sum_{n=1}^{\infty} \frac{2^n}{(2n)!}$

36.  $\sum_{n=1}^{\infty} \frac{1}{2^n+n}$

37.  $\sum_{n=1}^{\infty} \frac{n^3}{e^n}$

38.  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

39.  $\sum_{n=1}^{\infty} \frac{e^{-\sqrt{n}}}{\sqrt{n}}$

40.  $\sum_{n=1}^{\infty} \frac{2^n}{n(n+3)}$

41.  $\sum_{n=1}^{\infty} \frac{3n^4-2n^3+7}{5n^7+n^2-3n}$

42.  $\sum_{n=2}^{\infty} \frac{n+3}{(\ln n)^2}$

43.  $\sum_{n=1}^{\infty} \frac{2n-1}{\sqrt{n^5+2}}$

44.  $\sum_{n=1}^{\infty} \frac{5n^2}{(n+1)(n+2)(n+3)(n+4)}$

45.  $\sum_{n=1}^{\infty} \frac{1}{(2n+1)^{1/2}}$

46.  $\sum_{n=1}^{\infty} \frac{1}{n \ln^2 n}$

47.  $\sum_{n=1}^{\infty} \sin n\pi$
48.  $\sum_{n=1}^{\infty} \frac{n^2}{n^3 - 1000}$
49.  $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$
50.  $\sum_{n=1}^{\infty} \frac{n}{n+1}$
51.  $\sum_{n=1}^{\infty} \frac{n}{2^n}$
52.  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$
53.  $\sum_{n=1}^{\infty} \frac{\sqrt{n+2}}{n^3+1}$
54.  $\sum_{n=1}^{\infty} \frac{(n+1)!}{(n-1)!2^n}$
55.  $\sum_{n=1}^{\infty} \left(1 + \frac{4}{n}\right)^n$
56.  $\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$
57.  $\sum_{n=1}^{\infty} ne^{-n^2}$
58.  $\sum_{n=1}^{\infty} \frac{10n^3 + 7n^2}{n^{3/2} + 3}$
59.  $\sum_{n=1}^{\infty} \frac{3^n}{(2n)!}$
60.  $\sum_{n=1}^{\infty} \frac{7^n}{(3n)!}$
61.  $\sum_{n=1}^{\infty} \frac{1}{3 \cdot 2^n + 5}$
62.  $\sum_{n=1}^{\infty} \frac{n+2}{n\sqrt{5n-2}}$
63.  $\sum_{n=1}^{\infty} \frac{n^2 - 2n + 1}{(n-3)(n+2)}$
64.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{99+n^2}}$
65.  $\sum_{n=1}^{\infty} \frac{1}{(2 + \frac{1}{n})^n}$
66.  $\sum_{n=1}^{\infty} \frac{e^n}{n^3}$
67.  $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$
68.  $\sum_{n=1}^{\infty} \frac{n^2 + 3n + 1}{4n^3 + 1}$
69.  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{3/2}}$
70.  $\sum_{n=2}^{\infty} \frac{4}{3^n}$
71.  $\sum_{n=1}^{\infty} \frac{e^n}{n!}$
72.  $\sum_{n=1}^{\infty} \ln \left(\frac{n+1}{2n+1}\right)$
73.  $\sum_{k=1}^{\infty} \frac{k^2}{k!}$
74.  $\sum_{n=1}^{\infty} \frac{5n}{(n^7 + 3)^{1/3}}$
75.  $\sum_{k=2}^{\infty} \frac{1}{k\sqrt[3]{\ln k}}$
76.  $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2 + 4}$
77.  $\sum_{m=1}^{\infty} \frac{\arctan m}{1 + m^2}$
78.  $\sum_{n=1}^{\infty} \frac{n+3}{2^n}$
79.  $\sum_{n=1}^{\infty} \frac{n+2}{\sqrt{3n^3 + n + 1}}$
80.  $\sum_{n=1}^{\infty} \left\{\frac{1}{2} + \frac{2}{3n}\right\}^n$
81.  $\sum_{n=2}^{\infty} \left(\frac{\sqrt{n}}{\ln n} + (-1)^n \frac{1}{3^n}\right)$
82.  $\sum_{n=1}^{\infty} \frac{(n+2)!}{4^{n+1}}$
83.  $\sum_{n=1}^{\infty} n^2 e^{-n^3}$
84.  $\sum_{n=1}^{\infty} \frac{5\sqrt{n^2+2}}{3n+1}$
85.  $\sum_{n=1}^{\infty} \frac{2\sqrt{n} + 5n}{\sqrt{3n^3+4}}$
86.  $\sum_{n=3}^{\infty} \frac{e}{(3n+2)(2n-5)}$
87.  $\sum_{m=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2m-1)}{m!}$
88.  $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$
89.  $\sum_{n=1}^{\infty} \frac{\cos \frac{1}{n}}{\arctan n}$
90.  $\sum_{n=1}^{\infty} \frac{3n+2}{(n+1)(4n)}$
91.  $\sum_{n=1}^{\infty} \ln \left[\frac{n^2+2}{3n^2+5}\right]$
92.  $\sum_{n=1}^{\infty} \frac{e^n}{(2n+1)!}$
93.  $\sum_{n=1}^{\infty} \left[\frac{n+3}{2n+1}\right]^n$
94.  $\sum_{n=1}^{\infty} \frac{1}{2+5^n}$
95.  $\sum_{n=1}^{\infty} ne^{-n}$
96.  $\sum_{n=1}^{\infty} \cos(n\pi)$
97.  $\sum_{n=1}^{\infty} \frac{(n+1)!}{3^{2n+1}}$
98.  $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} + \frac{1}{5^n}\right)$
99.  $\sum_{n=1}^{\infty} \frac{(2n)!}{3^n}$
100.  $\sum_{n=1}^{\infty} \frac{3^{2n}}{1 + \ln n}$

$$101. \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+3}}$$

$$102. \sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$$

$$103. \sum_{n=1}^{\infty} \frac{4\sqrt{n}-1}{n^2+2\sqrt{n}}$$

$$104. \sum_{n=1}^{\infty} \ln\left[\frac{2n+1}{n}\right]$$

$$105. \sum_{n=1}^{\infty} \frac{3^n}{n5^n+2}$$

$$106. \sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$$

18. Diverges (Ratio Test)

19. Diverges (DT)

20. Converges (IT)

21. Diverges (geometric series)

22. Diverges (DT)

23. Converges (LCT)

24. Diverges ( $\lim S_n$ )

25. Diverges (DT)

26. Diverges (LCT)

27. Converges (IT)

28. Converges (Ratio Test)

29. Converges (LCT)

30. Diverges (DT)

31. Diverges (DT)

32. Converges (LCT)

33. Diverges (Ratio Test)

34. Diverges (IT)

35. Converges (Ratio Test)

36. Converges (CT)

37. Converges (Ratio Test)

38. Converges (IT)

39. Converges (IT)

40. Diverges (Ratio Test)

41. Converges (LCT)

42. Diverges (DT)

43. Converges (LCT)

44. Converges (LCT)

45. Diverges (LCT)

46. Converges (IT)

47. Converges ( $\lim S_n$ )

48. Diverges (LCT)

49. Diverges (IT)

50. Diverges (DT)

51. Converges (Ratio Test)

52. Converges (LCT)

53. Converges (LCT)

54. Converges (Ratio Test)

55. Diverges (DT)

56. Diverges ( $\lim S_n$ )

57. Converges (IT)

58. Diverges (DT)

59. Converges (Ratio Test)

60. Converges (Ratio Test)

61. Converges (CT)

62. Diverges (LCT)

63. Diverges (DT)

64. Diverges (LCT)

65. Converges (CT)

66. Diverges (Ratio Test)

67. Converges (Ratio Test)

68. Diverges (LCT)

69. Converges (IT)

70. Converges  
(geometric-series)

71. Converges (Ratio Test)

72. Diverges (DT)

73. Converges (Ratio Test)

74. Converges (LCT)

75. Diverges (IT)

76. Converges (LCT)

77. Converges (IT)

78. Converges (Ratio Test)

79. Diverges (LCT)

80. Converges (Root Test)

81. Diverges (DT + geometric series)

### Answers

1. Converges (LCT)

2. Diverges ( $p$ -series)

3. Diverges (LCT)

4. Converges (Ratio Test)

5. Converges (LCT)

6. Converges (Ratio Test)

7. Diverges ( $p$ -series)

8. Converges (LCT)

9. Converges (Ratio Test)

10. Converges (Ratio Test)

11. Diverges (DT)

12. Diverges (IT)

13. Converges (Ratio Test)

14. Converges (LCT)

15. Converges (Ratio Test)

16. Converges (LCT)

17. Diverges (DT)

- |                           |  |                           |
|---------------------------|--|---------------------------|
| 82. Diverges (Ratio Test) | 91. Diverges (DT)                          | 99. Diverges (Ratio Test) |
| 83. Converges (IT)        | 92. Converges (Ratio Test)                 | 100. Diverges (DT)        |
| 84. Diverges (DT)         | 93. Converges (Root Test)                  | 101. Converges (LCT)      |
| 85. Diverges (LCT)        | 94. Converges (CT)                         | 102. Diverges (LCT)       |
| 86. Converges (LCT)       | 95. Converges (Ratio Test)                 | 103. Converges (CT)       |
| 87. Diverges (LCT)        | 96. Diverges (DT)                          | 104. Diverges (DT)        |
| 88. Converges (IT)        | 97. Diverges (Ratio Test)                  | 105. Converges (CT)       |
| 89. Diverges (DT)         | 98. Diverges ( $p$ -series + geom. series) | 106. Diverges (DT)        |
| 90. Diverges (LCT)        |  |                           |