

1 Limits

Find the following limits, if they exist. If the limit does not exist, state the reason.

- $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 3x + 2}$
- $\lim_{x \rightarrow 0} \frac{\pi}{4}$
- $\lim_{x \rightarrow -\infty} \frac{-x^2 + 6x + 8}{10x^2 + 10^6}$
- $\lim_{x \rightarrow 3} \frac{\sqrt{x} - \sqrt{3}}{x - 3}$
- $\lim_{h \rightarrow 4} \frac{2x^2h - 2xh + 25h^2}{\sqrt{h} - 5}$
- $\lim_{h \rightarrow 0} \frac{\sqrt{2h+4} - 2}{h}$
- $\lim_{x \rightarrow 3^-} \frac{(2x+1)^3(x-5)^4}{(x-3)^3\sqrt{9-x}}$
- $\lim_{x \rightarrow 0} \frac{x^2 + 2x}{x^3 - 3x}$
- $\lim_{x \rightarrow \infty} \frac{2x^2 + 6x + 3}{8 + 3x - 5x^2}$
- $\lim_{x \rightarrow 0} \frac{\sqrt{3-x} - \sqrt{3}}{x}$
- $\lim_{x \rightarrow 2} \frac{x+3}{x-4}$
- $\lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{x^2 - 6x + 5}$
- $\lim_{x \rightarrow -\infty} \frac{3x^3 + 5x^2 - 7}{5x - 4x^3}$
- $\lim_{h \rightarrow 0} \frac{\sqrt{5+h} - \sqrt{5}}{h}$
- $\lim_{x \rightarrow 3^-} \frac{x^2|x-3|}{x-3}$
- $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x - 2}$
- $\lim_{x \rightarrow \infty} e^{-x} + 1$
- $\lim_{x \rightarrow \infty} \frac{2x^2 - 3x + 7}{x^3 - 3}$
- $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 + x - 6}$
- $\lim_{x \rightarrow -3^-} \frac{x-3}{x+3}$
- $\lim_{x \rightarrow \infty} \frac{x-x^2}{x+x^2}$
- $\lim_{x \rightarrow 0} \frac{x-x^2}{x+x^2}$
- $\lim_{x \rightarrow -7} \frac{x^2 + 5x - 14}{x+7}$
- $\lim_{x \rightarrow \infty} \frac{12 + 2^x}{6 - 2^x}$
- $\lim_{x \rightarrow \infty} \frac{2x - 5}{x^2 + x + 1}$
- $\lim_{x \rightarrow 4} \frac{x^2 - x - 12}{x - 4}$
- $\lim_{x \rightarrow -2} \frac{x^2 + 7x + 10}{x + 2}$
- $\lim_{x \rightarrow \infty} \frac{6x^3 + x^2 + 7x - 2}{5x^2 + 2x - 1}$
- $\lim_{x \rightarrow \infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$
- $\lim_{x \rightarrow 3^-} \frac{10x^3 - x^2}{x - 3}$
- $\lim_{x \rightarrow -2} (2x^2 - 6x + 5)^2$
- $\lim_{x \rightarrow -5} \frac{x+5}{x^2 + 4x - 5}$
- $\lim_{x \rightarrow 0} \frac{(x+2)^2 - 4}{x}$
- $\lim_{x \rightarrow 0^+} \pi$
- $\lim_{h \rightarrow \infty} -2h$
- $\lim_{y \rightarrow 2^-} \frac{(y-1)(y-2)}{y+1}$
- $\lim_{x \rightarrow 1^+} \frac{x^4 - 1}{x - 1}$
- $\lim_{x \rightarrow -\infty} \frac{x-2}{x^2 + 2x + 1}$
- $\lim_{x \rightarrow 0} \frac{x}{|x|}$
- $\lim_{h \rightarrow 0} \frac{\sin 2h}{h}$
- $\lim_{t \rightarrow 2} \frac{t^2 - t - 2}{t^2 - 3t + 2}$

42. $\lim_{h \rightarrow 3} \frac{h^2 + h + 1}{\sqrt{h + 6}}$
43. $\lim_{x \rightarrow 0} \frac{\sqrt{2 + x} - \sqrt{2 - x}}{x}$
44. $\lim_{x \rightarrow \infty} \frac{3x^4 - 2x^2 + 1}{7x^4 + 6x^3 + x}$
45. $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 2x + 2}}{x}$
46. $\lim_{x \rightarrow 2} \frac{x^2 - 2x + 3}{\sqrt{8x}}$
47. $\lim_{h \rightarrow 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h}$
48. $\lim_{x \rightarrow 1^+} \frac{x^2 + 2x - 3}{x^2 - 2x + 1}$
49. $\lim_{x \rightarrow \infty} \frac{x^2 - x^{-2}}{4x^2 + 4x^{-2}}$
50. $\lim_{r \rightarrow -1} \sqrt[3]{\frac{28 + r}{r}}$
51. $\lim_{x \rightarrow -\infty} \frac{x^2 + 6x + 5}{3x^2 + 4}$
52. $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$
53. $\lim_{x \rightarrow -2} \frac{(x + 2)(x^2 - 3x + 5)}{x^2 + 3x + 2}$
54. $\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x^2 - 6x + 8}$
55. $\lim_{x \rightarrow 0} \frac{\sqrt{x + 9} - 3}{x}$
56. $\lim_{x \rightarrow \infty} \frac{3x^3 - 4x^2 + 6x - 5}{3 + x - x^2 - 2x^3}$
57. $\lim_{x \rightarrow 0} \frac{x^3 - 1}{x - 1}$
58. $\lim_{x \rightarrow 0} \frac{\sqrt{x + 4} - 2}{x}$
59. $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x^2 - 4}$
60. $\lim_{x \rightarrow 0} \frac{2 - \sqrt{4 - x}}{3x}$
61. $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1}}{3x - 1}$
62. $\lim_{x \rightarrow \pi} \frac{2 \cos^2 x - 1}{\sin^2 x - 1}$
63. $\lim_{x \rightarrow 3} \frac{(x - 3)^2}{x^2 - 5x + 6}$
64. $\lim_{t \rightarrow \infty} \frac{1 - t^2 + 2t^3}{t - 5t^3}$
65. $\lim_{x \rightarrow 0} \frac{\sin^2 3x}{x^2}$
66. $\lim_{x \rightarrow 7^+} \frac{3x - 21}{x^2 - x - 42}$
67. $\lim_{x \rightarrow 0} \frac{3x}{\sin 5x}$
68. $\lim_{x \rightarrow -\infty} \frac{2 - 3x^2}{2x^2 + 5x - 7}$
69. $\lim_{x \rightarrow 3^-} \frac{9 - x^2}{x^2 - 6x + 9}$
70. $\lim_{y \rightarrow \pi/2} \frac{\sin y}{\cos y - 1}$
71. $\lim_{x \rightarrow 1} \frac{x^2 + 3x - 4}{x - 1}$
72. $\lim_{x \rightarrow 5^-} \frac{\sqrt{5 - x}}{x - 5}$
73. $\lim_{x \rightarrow 0} \frac{\sqrt{x + 2} - \sqrt{2}}{x}$
74. $\lim_{x \rightarrow \infty} \frac{x^2 + 3x}{1 - x^2 - x^3}$
75. $\lim_{x \rightarrow 6} \frac{2x^2 - 12x}{x^2 + x - 42}$
76. $\lim_{x \rightarrow -4} \frac{3 - \sqrt{25 - x^2}}{x + 4}$
77. $\lim_{x \rightarrow 0} \frac{x^2}{\sin^2 3x}$
78. $\lim_{x \rightarrow -\infty} \frac{x^3}{4 + x^2}$
79. $\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{x - 1}$
80. $\lim_{x \rightarrow 0} \frac{x}{\sin 4x}$
81. $\lim_{x \rightarrow \infty} \frac{2x^3 + 12x^2 - 18x}{x^3 - x - 12}$
82. $\lim_{x \rightarrow 5} \frac{x^2 - 3x - 10}{x^2 + 2x - 35}$

83. Given the function

$$f(x) = \begin{cases} \frac{8x^3+16x^2}{x+2} & \text{if } x < -\frac{1}{2} \\ 3 & \text{if } x = -\frac{1}{2} \\ \frac{\sin \pi x}{x} & \text{if } x > -\frac{1}{2} \end{cases}$$

Find the limits

- (a) $\lim_{x \rightarrow 0} f(x)$
- (b) $\lim_{x \rightarrow -\frac{1}{2}} f(x)$
- (c) $\lim_{x \rightarrow -2^+} f(x)$
- (d) $\lim_{x \rightarrow -\infty} f(x)$

84. Given the function

$$y = \frac{|x| - x}{x}$$

Find the limits

- (a) $\lim_{x \rightarrow 0^+} y$
- (b) $\lim_{x \rightarrow 0^-} y$
- (c) $\lim_{x \rightarrow 0} y$

85. $\lim_{x \rightarrow -3} \sqrt{\frac{x^2 - 4x + 4}{x^2 - 9x}}$

86. $\lim_{x \rightarrow 0} \frac{\sqrt{x+16} - 4}{x}$

87. $\lim_{x \rightarrow 1} \frac{\ln x \sin(x-1)}{x-1}$

88. $\lim_{x \rightarrow \infty} \frac{x^3 - 7x^4}{3x^4 - 5x^3}$

89. $\lim_{y \rightarrow A} \frac{\frac{1}{A^2} - \frac{1}{y^2}}{y - A}$

90. $\lim_{x \rightarrow 0} \frac{\sin 3x}{x \cos x}$

91. $\lim_{x \rightarrow \infty} \frac{\sqrt{10x^2 - 5x + 3}}{1 - 3x}$

92. $\lim_{x \rightarrow 2^-} \frac{(3x+4)^2}{x^2 - x - 2}$

93. $\lim_{x \rightarrow 2} \frac{4 - x^2}{x^2 - 3x + 2}$

94. Given the function

$$f(x) = \begin{cases} \frac{x^2-1}{x^2-x-2} & \text{if } x < 0 \\ 3 & \text{if } x = 0 \\ \frac{\sqrt{x-2}}{x-4} & \text{if } x > 0 \end{cases}$$

Evaluate the limits

- (a) $\lim_{x \rightarrow 0} f(x)$
- (b) $\lim_{x \rightarrow 4} f(x)$
- (c) $\lim_{x \rightarrow -1} f(x)$
- (d) $\lim_{x \rightarrow 2} f(x)$
- (e) $\lim_{x \rightarrow -\infty} f(x)$

95. $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 2x}$

96. $\lim_{x \rightarrow -\infty} \frac{4x+3}{3x^2+5}$

97. $\lim_{x \rightarrow -5^-} \frac{x^2 - 25}{x^2 + 10x + 25}$

98. $\lim_{x \rightarrow 3} \frac{\sqrt{25-x^2} - 4}{x-3}$

99. $\lim_{x \rightarrow -3} \frac{2x^2 + 5x - 3}{3x^2 + x - 24}$

100. $\lim_{x \rightarrow 0} \left[\frac{x}{3 \sin x} + \frac{\sin x}{3x} \right]$

101. $\lim_{x \rightarrow -\infty} \left[e^x + \frac{2x}{x+1} \right]$

102. $\lim_{x \rightarrow 4^+} \frac{2-x}{(x-4)^2}$

Answers:

1. 3
2. $\frac{\pi}{4}$
3. $-\frac{1}{10}$
4. $\frac{1}{2\sqrt{3}}$
5. $\frac{8x^2-8x+400}{-3}$
6. $\frac{1}{2}$
7. $-\infty$
8. $-\frac{2}{3}$
9. $-\frac{2}{5}$
10. $-\frac{1}{2\sqrt{3}}$
11. $-\frac{5}{2}$
12. $-\frac{3}{4}$
13. $-\frac{3}{4}$
14. $\frac{1}{2\sqrt{5}}$
15. -9
16. 7
17. 1
18. 0
19. $\frac{1}{5}$
20. ∞
21. -1
22. 1
23. -9
24. -1
25. 0
26. 7
27. 3
28. ∞
29. 1
30. $-\infty$
31. 625
32. $-\frac{1}{6}$
33. 4
34. π
35. $-\infty$
36. 0
37. 4
38. 0
39. Does not exist
40. 2
41. 3
42. $\frac{13}{3}$
43. $\frac{1}{\sqrt{2}}$
44. $\frac{3}{7}$
45. -1
46. $\frac{3}{4}$
47. $-\frac{1}{25}$
48. ∞
49. $\frac{1}{4}$
50. -3
51. $\frac{1}{3}$
52. $\frac{1}{6}$
53. -15
54. 0
55. $\frac{1}{6}$
56. $-\frac{3}{2}$
57. 1
58. $\frac{1}{4}$
59. $\frac{7}{4}$
60. $\frac{1}{12}$
61. $-\frac{1}{3}$
62. -1
63. 0
64. $-\frac{2}{5}$
65. 9

66. $\frac{3}{13}$
67. $\frac{3}{5}$
68. $-\frac{3}{2}$
69. ∞
70. -1
71. 5
72. $-\infty$
73. $\frac{1}{2\sqrt{2}}$
74. 0
75. $\frac{12}{13}$
76. $-\frac{4}{3}$
77. $\frac{1}{9}$
78. $-\infty$
79. $-\frac{1}{2}$
80. $\frac{1}{4}$
81. 2
82. $\frac{7}{12}$
83. (a) π
 (b) 2
 (c) 32
 (d) ∞
84. (a) 0
 (b) -2
 (c) Does not exist
85. $\frac{5}{6}$
86. $\frac{1}{8}$
87. 0
88. $-\frac{7}{3}$
89. $\frac{2}{A^3}$
90. 3
91. $-\frac{\sqrt{10}}{3}$
92. $-\infty$
93. -4
94. (a) $\frac{1}{2}$
- (b) $\frac{1}{4}$
- (c) $\frac{2}{3}$
- (d) $\frac{2-\sqrt{2}}{2}$
- (e) 1
95. $\frac{3}{2}$
96. 0
97. ∞
98. $-\frac{3}{4}$
99. $\frac{7}{17}$
100. $\frac{2}{3}$
101. 2
102. $-\infty$