

From the files of Norman Dobson
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Calculus II – Final Exam Problems
Integration

Evaluate each integral.

1. $\int \frac{x}{\sqrt{x^2 - 1}} dx$
2. $\int x \cos ax dx$
3. $\int \frac{7x + 1}{x^2 - x - 2} dx$
4. $\int_0^{\pi/2} \cos^3 x dx$
5. $\int \frac{x^3 dx}{\sqrt{x + 1}}$
6. $\int_0^{\infty} \frac{dx}{(x + 1)^2}$
7. $\int_1^5 \frac{x^2 + 1}{\sqrt{x - 1}} dx$
8. $\int \sin^6 x \cos^3 x dx$
9. $\int \sin \theta \ln(\cos \theta) d\theta$
10. $\int_1^2 \frac{dx}{\sqrt{x - 1}}$
11. $\int_2^{\infty} \frac{dx}{(x - 1)^2}$
12. $\int \sec^3 2\theta \tan^5 2\theta d\theta$
13. $\int_1^2 x^2 \sqrt{2 - x} dx$
14. $\int \frac{dx}{x(x + 1)^2}$
15. $\int_0^{\sqrt{3}} \frac{dx}{(4 - x^2)^{3/2}}$
16. $\int_{-2}^2 \frac{dx}{(x + 2)^2}$
17. $\int \frac{1}{x(x + 2)} dx$
18. $\int \arctan \sqrt{x} dx$
19. $\int_1^{\infty} e^{-x} dx$
20. $\int \frac{x^3}{\sqrt{4 - x^2}} dx$
21. $\int_0^1 \frac{dx}{x}$
22. $\int_1^5 (x + 1)\sqrt{2x - 1} dx$
23. $\int \sin^5 2\theta \cos^3 2\theta d\theta$
24. $\int_0^{\sqrt{2}} \frac{x^3}{\sqrt{4 - x^2}} dx$
25. $\int x \arctan x dx$
26. $\int_0^3 \frac{dx}{(x - 1)^2}$
27. $\int e^x \sin x dx$
28. $\int x^2 \sqrt{x + 1} dx$
29. $\int x \sin^3 x^2 \cos^4 x^2 dx$
30. $\int \cot^3 2x \csc^3 2x dx$
31. $\int_{-1}^2 \frac{dx}{x^2}$
32. $\int_1^{\infty} \frac{dx}{(2x - 1)^{3/2}}$
33. $\int 4 \cos^3\left(\frac{x}{3}\right) \sin^2\left(\frac{x}{3}\right) dx$
34. $\int_0^{\pi/6} 4 \sin^2(3x) dx$
35. $\int \sec^2 x \ln |\cot x| dx$
36. $\int \sec^4 x \tan^3 x dx$
37. $\int \frac{x^2}{(9 - 4x^2)^{3/2}} dx$
38. $\int_6^{\infty} \frac{dx}{x\sqrt{x^2 - 9}}$
39. $\int_0^5 \frac{dx}{x^2 - 4x + 4}$
40. $\int \sin^2 x \cos^2 x dx$
41. $\int \cot^3 x dx$
42. $\int \sec^6 x \tan^3 x dx$
43. $\int_0^{\pi/4} \sin x \ln(\sec x) dx$
44. $\int \frac{\sqrt{9x^2 - 4}}{x} dx$
45. $\int_{2\sqrt{3}}^{\infty} \frac{dx}{x^2 + 4}$
46. $\int_0^1 \ln x dx$
47. $\int (\tan 2x)^2 \sec 2x dx$
48. $\int \frac{dx}{x^2 \sqrt{x^2 - 1}}$
49. $\int x^2 (\ln x)^2 dx$
50. $\int_0^{\pi/4} \frac{\sin^3 x}{\cos^4 x} dx$
51. $\int_0^{\infty} \frac{1}{\sqrt{e^x}} dx$
52. $\int_0^4 \frac{1}{(x - 2)^2} dx$
53. $\int \sin^3 x \cos^4 x dx$
54. $\int \frac{dx}{e^x + e^{-x}}$
55. $\int x e^{-x} dx$

56. $\int \cos^2(\ln x) \frac{dx}{x}$
57. $\int e^{2x} \cos 3x \, dx$
58. $\int x^3 \sqrt{x^2 - 4} \, dx$
59. $\int_{-\infty}^{\infty} \frac{dx}{4 + x^2}$
60. $\int_0^2 \frac{dx}{(x-1)^2}$
61. $\int \frac{\sec^2 3x}{\tan 3x} \, dx$
62. $\int \frac{x^3}{\sqrt{4x^2 - 9}} \, dx$
63. $\int_{-2}^2 \frac{dx}{x^3}$
64. $\int_0^{\infty} x e^{-x} \, dx$
65. $\int \frac{5x^2 - 11x + 5}{(x-1)^2(x-2)} \, dx$
66. $\int \ln^2 x \, dx$
67. $\int \frac{x^5}{\sqrt[3]{x^3 + 1}} \, dx$
68. $\int \sin^3 2\theta \cos 2\theta \, d\theta$
69. $\int_{-1}^1 \frac{(\ln|x|)^3}{x} \, dx$
70. $\int_{\sqrt{5}}^{\sqrt{20}} \frac{dx}{(x^2 - 4)^{3/2}}$
71. $\int_{-\infty}^0 x e^{2x} \, dx$
72. $\int x \tan^2 x \, dx$
73. $\int \tan^4 x \, dx$
74. $\int \sqrt{x^2 + 9} \, dx$
75. $\int \sqrt{x} \cos \sqrt{x} \, dx$
76. $\int_{-1}^1 \frac{dx}{(2x+1)^3}$
77. $\int_e^{e^2} \ln x \sqrt{x \ln x - x} \, dx$
78. $\int_{-\infty}^{\infty} x e^{-|x|} \, dx$
79. $\int \csc^3 x \cot x \, dx$
80. $\int \sin^4\left(\frac{x}{4}\right) \, dx$
81. $\int_0^{3/2} \frac{dx}{\sqrt{9 - 2x^2}}$
82. $\int \arcsin x \, dx$
83. $\int \sec^3 \theta \, d\theta$
84. $\int \frac{x^3}{\sqrt{x^2 - 4}} \, dx$
85. $\int_0^{\infty} (x+1)e^{-x} \, dx$
86. $\int \tan^3 2x \sec^4 2x \, dx$
87. $\int_{1/\sqrt{2}}^{\sqrt{3}/2} \frac{\arccos x}{\sqrt{1-x^2}} \, dx$
88. $\int \frac{\tan x}{\ln \sec x} \, dx$
89. $\int e^x \sin 2x \, dx$
90. $\int_0^1 \frac{t^2}{(25 - 9t^2)^{3/2}} \, dt$
91. $\int \frac{\arctan x}{x^2} \, dx$
92. $\int \frac{\sin x e^{\sec x}}{\cos^2 x} \, dx$
93. $\int_1^e x^3 \ln x \, dx$
94. $\int (\arcsin x)^2 \, dx$
95. $\int_0^2 \frac{dx}{(x-1)^{2/3}}$
96. $\int \tan^3 2x \sec^3 2x \, dx$
97. $\int \frac{3x-2}{(x+1)^2(x-1)} \, dx$
98. $\int_{1/\sqrt{2}}^{\sqrt{2}} (1-t)\sqrt{2-t^2} \, dt$
99. $\int_1^{\infty} \frac{\arctan x}{x^2 + 1} \, dx$
100. $\int_1^{\sqrt{2}} x \operatorname{arcsec} x \, dx$
101. $\int e^{x/2} \cos 3x \, dx$
102. $\int \frac{\sec^4 x}{\sqrt{\tan x}} \, dx$
103. $\int \frac{x^2}{x^4 - 1} \, dx$
104. $\int_0^1 \frac{dx}{(3x-1)^{2/3}}$
105. $\int_1^{\infty} \frac{dx}{x^2} - \int_0^1 \frac{dy}{y^{1/3}}$
106. $\int_0^{10} x e^{5x} \, dx$

Answers:

1. $\sqrt{x^2 - 1} + C$
2. $\frac{\cos ax}{a^2} + \frac{x \sin ax}{a} + C$
3. $2 \ln|x + 1| + 5 \ln|x - 2| + C$
4. $\frac{2}{3}$
5. $\frac{2}{7}(x + 1)^{7/2} - \frac{6}{5}(x + 1)^{5/2} + 2(x + 1)^{3/2} - 2\sqrt{x + 1} + C$
6. 1 (Improper)
7. $\frac{472}{15}$ (Improper)
8. $\frac{1}{7} \sin^7 x - \frac{1}{9} \sin^9 x + C$
9. $[1 - \ln|\cos \theta|] \cos \theta + C$
10. 2 (Improper)
11. 1 (Improper)
12. $\frac{1}{14} \sec^7 2\theta - \frac{1}{5} \sec^5 2\theta + \frac{1}{6} \sec^3 2\theta + C$
13. $\frac{142}{105}$
14. $\ln|x| + \frac{1}{x+1} - \ln|x + 1| + C$
15. $\frac{\sqrt{3}}{4}$
16. Diverges. (Improper)
17. $\frac{1}{2} \ln|x| - \frac{1}{2} \ln|x + 2| + C$
18. $(x + 1) \arctan \sqrt{x} - \sqrt{x} + C$
19. $\frac{1}{e}$ (Improper)
20. $\frac{1}{3}(4 - x^2)^{3/2} - 4\sqrt{4 - x^2} + C$
21. Diverges. (Improper)
22. $\frac{186}{5}$
23. $\frac{1}{12} \sin^6 2\theta - \frac{1}{16} \sin^8 2\theta + C$
24. $\frac{2}{3}(8 - 5\sqrt{2})$
25. $\frac{1}{2}(x^2 + 1) \arctan x - \frac{1}{2}x + C$
26. Diverges. (Improper)
27. $\frac{1}{2}e^x(\sin x - \cos x) + C$
28. $\frac{2}{7}(x + 1)^{7/2} - \frac{4}{5}(x + 1)^{5/2} + \frac{2}{3}(x + 1)^{3/2} + C$
29. $-\frac{1}{10} \cos^5 x^2 + \frac{1}{14} \cos^7 x^2 + C$
30. $-\frac{1}{10} \csc^5 2x + \frac{1}{6} \csc^3 2x + C$
31. Diverges. (Improper)
32. 1 (Improper)
33. $3 \sin^3(\frac{x}{3}) - 2 \sin^5(\frac{x}{3}) + C$
34. $\frac{\pi}{3}$
35. $[\ln|\cot x| + 1] \tan x + C$
36. $\frac{1}{4} \tan^4 x + \frac{1}{6} \tan^6 x + C$
37. $\frac{1}{4} \frac{x}{\sqrt{9-4x^2}} - \frac{1}{8} \arcsin \frac{2x}{3} + C$
38. $\frac{\pi}{18}$ (Improper)
39. Diverges. (Improper)
40. $\frac{x}{8} - \frac{1}{32} \sin 4x + C$
41. $-\frac{1}{2} \cot^2 x + \ln|\csc x| + C$
42. $\frac{1}{4} \tan^4 x + \frac{1}{3} \tan^6 x + \frac{1}{8} \tan^8 x + C$
43. $1 - \frac{1}{\sqrt{2}}(1 + \ln \sqrt{2})$
44. $\sqrt{9x^2 - 4} - 2 \operatorname{arcsec} \frac{3x}{2} + C$
45. $\frac{\pi}{12}$ (Improper)
46. -1 (Improper)
47. $\frac{1}{4}(\sec 2x \tan 2x - \ln|\sec 2x + \tan 2x|) + C$
48. $\frac{\sqrt{x^2-1}}{x} + C$
49. $\frac{x^3}{27}[9 \ln^2 x - 6 \ln x + 2] + C$
50. $\frac{2-\sqrt{2}}{3}$
51. 2 (Improper)
52. Diverges. (Improper)
53. $\frac{1}{7} \cos^7 x - \frac{1}{5} \cos^5 x + C$
54. $\arctan e^x + C$
55. $-e^{-x}(x + 1) + C$
56. $\frac{1}{4}[2 \ln x + \sin(2 \ln x)] + C$
57. $\frac{2}{13}e^{2x} \cos 3x + \frac{3}{13}e^{2x} \sin 3x + C$
58. $\frac{4}{3}(x^2-4)^{3/2} + \frac{1}{5}(x^2-4)^{5/2} + C$
59. $\frac{\pi}{2}$ (Improper)
60. Diverges. (Improper)
61. $\frac{1}{3} \ln|\tan 3x| + C$
62. $\frac{9}{16}\sqrt{4x^2 - 9} + \frac{1}{48}(4x^2 - 9)^{3/2} + C$
63. Diverges. (Improper)
64. 1 (Improper)
65. $-\frac{1}{x-1} + 2 \ln|x - 1| + 3 \ln|x - 2| + C$
66. $x \ln^2 x - 2x \ln x + 2x + C$
67. $\frac{1}{2}x^3(x^3 + 1)^{2/3} - \frac{3}{10}(x^3 + 1)^{5/3} + C$
68. $\frac{1}{8} \sin^4 2\theta + C$
69. Diverges. (Improper)
70. $\frac{\sqrt{5}}{8}$
71. $-\frac{1}{4}$ (Improper)
72. $x \tan x - \frac{1}{2}x^2 - \ln|\sec x| + C$
73. $x + \frac{1}{3} \tan^3 x - \tan x + C$
74. $\frac{1}{2}x\sqrt{x^2 + 9} + \frac{9}{2} \ln|\sqrt{x^2 + 9} + x| + C$
75. $2x \sin \sqrt{x} - 4 \sin \sqrt{x} + 4\sqrt{x} \cos \sqrt{x} + C$
76. Diverges. (Improper)
77. $\frac{2e^3}{3}$
78. 0 (Improper)
79. $-\frac{1}{3} \csc^3 x + C$
80. $\frac{3x}{8} - \sin \frac{x}{2} + \frac{1}{8} \sin x + C$
81. $\frac{\pi}{4\sqrt{2}}$
82. $x \arcsin x + \sqrt{1 - x^2} + C$
83. $\frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln|\sec \theta + \tan \theta| + C$
84. $4\sqrt{x^2 - 4} + \frac{1}{3}(x^2 - 4)^{3/2} + C$
85. 2
86. $\frac{1}{8} \tan^4 2x + \frac{1}{12} \tan^6 2x + C$
87. $\frac{5\pi^2}{288}$
88. $\ln \ln \sec x + C$

89. $\frac{1}{5}e^x(\sin 2x - 2 \cos 2x) + C$ 95. 6 (Improper) 101. $\frac{2}{37}e^{x/2}(\cos 3x + 6 \sin 3x) + C$
90. $\frac{1}{27} \left[\frac{3}{4} - \arcsin \frac{3}{5} \right]$ 96. $\frac{1}{10} \sec^5 2x - \frac{1}{6} \sec^3 2x + C$ 102. $2\sqrt{\tan x} + \frac{2}{5}(\tan x)^{5/2} + C$
91. $-\frac{1}{x} \arctan x + \ln|x| - \frac{1}{2} \ln(x^2 + 1) + C$ 97. $-\frac{5}{2(x+1)} - \frac{1}{4} \ln|x+1| + \frac{1}{4} \ln|x-1| + C$ 103. $\frac{1}{4} \ln|x-1| - \frac{1}{4} \ln|x+1| + \frac{1}{2} \arctan x + C$
92. $e^{\sec x} + C$ 98. $\frac{\pi}{3} - \frac{\sqrt{3}}{4}(1 + \sqrt{2})$ 104. $1 + \sqrt[3]{2}$ (Improper)
93. $\frac{1}{16}(3e^4 + 1)$ 99. $\frac{3\pi^2}{32}$ (Improper) 105. $-\frac{1}{2}$ (Improper)
94. $x(\arcsin x)^2 + 2\sqrt{1-x^2} \arcsin x - 2x + C$ 100. $\frac{1}{4}(\pi - 2)$ 106. $\frac{1}{25}(49e^{50} + 1)$