Linear Algebra 2

General Information.
Discipline: Mathematics *Course code*: 201-DDC-05
Ponderation: 3-2-3 *Credits*: $2\frac{2}{3}$ *Prerequisite*: 201-NYC-05
Objectives:

- OOUU: To apply knowledge and skills that have already been acquired to one or more topics in the natural sciences

Students are strongly advised to seek help from their instructor as soon as they encounter difficulties in the course.

Introduction. Linear Algebra 2 is an option course in the Science Program designed primarily for students who intend to follow a university program in Mathematics, Physics, or Engineering. It is normally taken in the fourth semester.

The branch of mathematics known as linear algebra is centered around three main ideas. The first of these is the theory of linear systems and this was the main topic covered in the first semester of linear algebra. Linear Algebra 2 will introduce the other two main concepts of linear algebra: (1) the theory of eigenvectors and eigenspaces, and (2) orthogonal vectors and projections. These two ideas are absolutely fundamental in most of the important applications of linear algebra in physics, chemistry, and all types of engineering.

The main types of objects dealt with in this course are the same as those of Linear Algebra 1, namely matrices and vectors. Ideas introduced in the first semester of linear algebra will be developed further and extended into new regions. The emphasis in this course will be on the proof of relevant theoretical results and on the practical applications of these results. The course has a strong interdisciplinary emphasis, we look at applications such as modelling populations, fitting curves to experimental data, and data compression techniques.

OBJECTIVES	STANDARDS
Statement of the Competency OOUU: To apply acquired knowledge to one or more subjects in the sciences. **Elements of the Competency** 1. To understand complex numbers. 2. To represent vectors with respect to various bases. 3. To diagonalize a square matrix. 4. To calculate orthogonal projections. 5. To understand the properties of symmetric matrices. 6. To produce the singular value decomposition of a matrix. 7. To produce the primary decomposition of a matrix.	**General Performance Criteria:** 1. Consistency and rigour in problem solving and justification of the approach used. 2. Observance of the experimental method. 3. Clarity and precision in oral and written communication. 4. Correct use of appropriate data-processing technology. 5. Appropriate choice of documents or lab instruments. 6. Significant contribution to the team. 7. Appropriate connections between science, technology and social progress.

OBJECTIVES	STANDARDS
Specific Performance Criteria	**Intermediate Learning Objectives ("A student is expected to …")**
1.0 **Complex Numbers.**	
1.1 Definition and Application	1.1.1 State the definition of the field of complex numbers. 1.1.2 Interpret complex numbers geometrically. 1.1.3 Find rectangular and polar forms. 1.1.4 Find real and imaginary parts. 1.1.5 Find roots of unity in the complex plane. 1.1.6 Find the conjugate of a complex number. 1.1.7 Solve various problems by applying the above definitions.
2.0 **Representing vectors in various bases.**	
2.1 Determination of coordinates relative to a basis	2.1.1 Find the coordinates of a vector relative to a specific basis. 2.1.2 Find the change of basis matrix to the standard basis. 2.1.3 Find the change of basis matrix between non-standard bases.
3.0 **Diagonalization.**	
3.1 Eigenvalues and Eigenvectors	3.1.1 State the definition of the terms eigenvalue and eigenvector. 3.1.2 Find the characteristic polynomial. 3.1.3 Calculate eigenvalues of a real matrix. 3.1.4 Find a basis for an eigenspace of a matrix. 3.1.5 Prove various properties of eigenvalues and eigenvectors.
3.2 Similarity	3.2.1 State the definition of similar matrices. 3.2.2 Diagonalize a matrix by eigenvectors. 3.2.3 Connect similarity with linear transformations. 3.2.4 Prove various properties of similar matrices. 3.2.5 Find the complex eigenvalues of a real matrix. 3.2.6 Represent 2×2 real matrices with complex eigenvalues as scalings and rotations.
3.3 Dynamical Systems	3.3.1 State the definition of a dynamical system. 3.3.2 Model dynamical systems by difference and/or differential equations. 3.3.3 Predict the long term behaviour of a dynamical system from eigenvalues. 3.3.4 Draw the phase plot of a system. 3.3.5 Categorize phase plots by the corresponding eigenvalues.
4.0 **Finding orthogonal projections.**	
4.1 Definition and Properties	4.1.1 State the definition of an inner product on $\mathbb{R}^n$. 4.1.2 Compute inner products. 4.1.3 Find length and distance from inner products. 4.1.4 State the definition of orthogonal sets of vectors. 4.1.5 Use the Gram-Schmidt procedure to convert a basis to an orthogonal basis. 4.1.6 Find a QR factorization of a matrix. 4.1.7 State the definition of an orthogonal matrix. 4.1.8 Compute orthogonal projections onto subspaces. 4.1.9 State the definition of the orthogonal complement of a subspace. 4.1.10 Find the orthogonal decomposition of a vector. 4.1.11 Prove various properties of projections and orthogonal matrices.
4.2 Finding Least Squares Approximations	4.2.1 State the definition of the least squares solution of a linear system. 4.2.2 State the definition of the normal equations. 4.2.3 State the definition of the best approximation of a vector relative to a subspace. 4.2.4 Find the least squares solution of a linear system. 4.2.5 Find the best approximation to a vector in a subspace. 4.2.6 Compute the least squares error. 4.2.7 Use a QR factorization to find the least squares solution. 4.2.8 Fit a least squares line to data. 4.2.9 Fit other curves to data in the least-squares sense.

OBJECTIVES	STANDARDS
Specific Performance Criteria	**Intermediate Learning Objectives**
4.3 Working with Inner Product Spaces	4.3.1 State the definition of an inner product space. 4.3.2 Compute inner products in a variety of inner product spaces. 4.3.3 Test for orthogonality in inner product spaces. 4.3.4 Find best approximations and corresponding errors in inner product spaces. 4.3.5 Find the Fourier series of a (periodic) function.
5.0 **Symmetric Matrices.**	
5.1 Understanding the properties of symmetric matrices	5.1.1 State the definition of symmetric and skew-symmetric matrices. 5.1.2 Prove some properties of symmetric and skew-symmetric matrices. 5.1.3 Find an orthogonal set of eigenvectors of a symmetric matrix. 5.1.4 Orthogonally diagonalize a symmetric matrix. 5.1.5 Compute the spectral decomposition of a symmetric matrix.
5.2 Working with Quadratic Forms	5.2.1 State the definition of a quadratic form. 5.2.2 Find the matrix representation of a quadratic form. 5.2.3 Eliminate the cross product terms from a quadratic form. 5.2.4 Classify quadratic forms as positive definite, indefinite, etc. 5.2.5 Find the maximum/minimum of a quadratic form with constraints.
6.0 **Singular Value Decomposition (SVD).**	
6.1 Definition and Application	6.1.1 State the definition of a SVD of a matrix. 6.1.2 Compute a SVD of a matrix. 6.1.3 Use a SVD to find the pseudoinverse of a matrix. 6.1.4 Interpret the SVD geometrically. 6.1.5 State the definition of the covariance matrix of a set of data. 6.1.6 Find the principal components of a set of multivariable data. 6.1.7 Use principal components to reduce the dimension of a set of multivariable data.
7.0 **Primary Decomposition.**	
7.1 The Minimum Polynomial	7.1.1 State the Cayley-Hamilton Theorem. 7.1.2 State the definition of the minimum polynomial of a square matrix. 7.1.3 State some properties of the minimum polynomial. 7.1.4 Find the minimum polynomial of a square matrix. 7.1.5 Use the minimum polynomial to determine whether a square matrix is diagonalizable.
7.2 The Primary Decomposition Theorem	7.2.1 State the definition of sums and intersections of subpaces. 7.2.2 State the definition of direct sums of subspaces. 7.2.3 State the definition of invariant subspaces relative to a linear transformation. 7.2.4 State the Primary Decomposition Theorem. 7.2.5 Express a square matrix in block diagonal form. 7.2.6 Find the Jordan Form of a square matrix.

Teaching Methods. This course will be 75 hours, meeting three times per week for a total of five hours per week. The course will rely mainly on the lecture method. The following methods may also be used: question-and-answer sessions, computer lab sessions, problem-solving sessions, class discussions, and assigned reading for independent study.

The use of cell phones, laptops, or similar technology for any purpose that is not directly related to the course is not permitted.

Other Resources.

Math Website.
http://departments.johnabbott.qc.ca/departments/mathematics

Math Lab. Located in H-022; open from 9:00 to 16:00 (weekdays) as a study area, and from 11:30 to 16:00 for borrowing course materials or using the computers and printers for math assignments.

Departmental Attendance Policy. Regular attendance is expected. Missing six classes is grounds for automatic failure in this course. Many of the failures in this course are due to students missing classes.

Required Text. *Linear Algebra and its Applications* by David C. Lay (3rd edition or newer). Available online for between $30 (3rd edition, used) and $180 (5th edition, new). Your teacher might have used copies for sale.

Course Costs. In addition to the cost of the textbook (see above), your instructor might recommend you acquire an inexpensive scientific calculator ($15-$25). *No calculators are allowed during tests or the final exam.*

Evaluation. The *Final Grade* will be the better of the following two options:

- 50% Final Exam plus 50% Class Mark
- 75% Final Exam plus 25% Class Mark

The Class Mark will include results from three or more tests (worth 75% of the Class Mark), and homework, quizzes or other assignments/tests (worth 25% of the Class Mark). The specifics of the Class Mark will be given by each instructor during the first week of classes in an appendix to this outline. The Final Exam is set by the Course Committee (which consists of all instructors currently teaching this course), and is marked by each individual instructor.

A student whose Class Mark is less than 50% may choose not to take the Final Exam in which case their grade will be their Class Mark or 50%, whichever is less.

Students must be available until the end of the final examination period to write exams.

College Policies. Article numbers refer to the IPESA (Institutional Policy on the Evaluation of Student Achievement, available at http://johnabbott.qc.ca/ipesa). Students are encouraged to consult the IPESA to learn more about their rights and responsibilities.

Changes to Evaluation Plan in Course Outline (Article 4.3). Changes to the evaluation plan, during the semester, require unanimous consent.

Mid-Semester Assessment MSA (Article 3.3). Students will receive an MSA in accordance with College procedures.

Religious Holidays (Article 3.2). Students who wish to observe religious holidays must inform their teacher in writing within the first two weeks of the semester of their intent.

Grade Reviews (Article 3.2, item 19). It is the responsibility of students to keep all assessed material returned to them in the event of a grade review. (The deadline for a Grade Review is 4 weeks after the start of the next regular semester.)

Results of Evaluations (Article 3.3, item 7). Students have the right to receive the results of evaluation, for regular day division courses, within two weeks. For evaluations at the end of the semester/course, the results must be given to the student by the grade submission deadline.

Cheating and Plagiarism (Articles 8.1 & 8.2). Cheating and plagiarism are serious infractions against academic integrity, which is highly valued at the College; they are unacceptable at John Abbott College. Students are expected to conduct themselves accordingly and must be responsible for all of their actions.