JOHN ABBOTT CEGEP/COLLEGE

Calculus III

General Information.
Discipline: Mathematics *Course code*: 201-DDB-05
Ponderation: 3-2-3 *Credits*: $2\frac{2}{3}$
Prerequisite: 201-NYB-05 (grade$>$ 65%)
Objectives:

- OOUV: To apply a scientific or technological approach to a field in the natural sciences
- OOUU: To apply knowledge and skills that have already been acquired to one or more topics in the natural sciences

Students are strongly advised to seek help from their instructor as soon as they encounter difficulties in the course.

Introduction. Calculus III is the last course in the Calculus sequence. It is offered as an option course in the Science Program for students intending to pursue university studies primarily in Engineering, Physics, Mathematics or Chemistry. It is normally taken in the fourth semester.

Calculus III extends the basic concepts of Calculus I and Calculus II (limits, continuity, derivatives and integrals) to vector-valued functions and to functions of two or more variables. It reinforces the student's skills in techniques of sketching graphs in two and three dimensions using parametric equations, polar, cylindrical and spherical coordinates. Geometric concepts from Linear Algebra are widely used. Calculus III also advances the treatment of power series begun in Calculus II. Emphasis will be placed on clarity and rigour in reasoning and in the application of methods. Some of the applied problems solved in this course will be taken from other disciplines in the program, primarily Physics.

Students are encouraged to use a scientific graphing calculator and suitable mathematical software programs (such as Maple) that are available for their use in the Mathematics Lab. However; only non- graphical calculators are permitted on tests and exams. The course uses a standard college-level Calculus textbook, chosen in collaboration with the Calculus I and II course committees.

Required Text. The textbook required for this course is

Essential Calculus, Multivariable (Custom Edition),
by James Stewart.

It is available at the bookstore for about $125.

Course Costs. In addition to the cost of the text listed above, a scientific calculator ($10 - $15) may be helpful. According to departmental policy, only the following models of calculators may be used for quizzes, tests, and the final exam: Sharp EL531WB-BK, TI-30XIIS, Casio FX-300MS Plus, and VICTOR 930-2. A graphics calculator ($100 - $150) could also be useful, but will not be allowed in tests or the exam.

OBJECTIVES	STANDARDS
Statement of the competency To apply the methods of multivariable calculus to the study of functions and problem solving.	**General Performance Criteria** • Appropriate choice of concepts, laws and principles. • Rigorous application of concepts, laws and principles. • Appropriate use of terminology. • Adequate graphical or mathematical representation. • Consistency and rigour in problem solving and justification of the approach used. • Observance of the scientific method and, where applicable, experimental procedure. • Justification of the approach used. • Assessment of the plausibility of results. • Correct algebraic operations. • Accuracy of calculations.
Elements of the Competency 1. To analyze the convergence of power series. 2. To describe plane curves using parametric equations and polar coordinates. 3. To apply calculus to vector-valued functions. 4. To make three dimensional drawings. 5. To study calculus of functions of several variables. 6. To solve optimization problems. 7. To calculate and apply double and triple integrals. 8. To undertake an interdisciplinary project which integrates current learning and demonstrates competence in three specific goals of the exit profile at an advanced level (00UU).	**Specific Performance Criteria** *[Specific performance criteria for each of these elements of the competency are listed on the following pages.]*

OBJECTIVES	STANDARDS
Specific Performance Criteria	**Intermediate Learning Objectives**
1. *Power series*	
1.1 Use of Taylor's Theorem to define functions	1.1.1. Write Taylor polynomials of a function centred at c. 1.1.2. Write Maclaurin polynomials of a function. 1.1.3. Write the Lagrange form of remainder. 1.1.4. Calculate bounds on errors. 1.1.5. Determine the accuracy of an approximation.
1.2 Analysis of convergence of power series	1.2.1. Define a power series centred at c. 1.2.2. Determine the radius and interval of convergence of a power series. 1.2.3. Determine convergence at the endpoints. 1.2.4. Differentiate power series. 1.2.5. Integrate power series.
1.3 Representing functions by power series	1.3.1. Recognize and use geometric power series. 1.3.2. Add power series. 1.3.3. Multiply power series. 1.3.4. Divide power series. 1.3.5. Substitute kx or x^n for the argument of a power series.
1.4 Use of Taylor/Maclaurin series to define a function	1.4.1. Define a Taylor/Maclaurin series. 1.4.2. Determine when a function equals its Taylor/Maclaurin series. 1.4.3. Recognize and use binomial series.
2. *Representation of plane curves*	
2.1 Use of parametric equations to analyze curves	2.1.1. Parametrize a variety of curves. 2.1.2. Recognize the orientation of a parametrized curve. 2.1.3. Determine when a parametrized curve is smooth or piecewise-smooth. 2.1.4. Determine dy/dx and d^2y/dx^2 for a parametrized curve.
2.2 Applications of parametric equations	2.2.1. Determine the arc length of a parametrized curve. 2.2.2. Determine the area of a surface of revolution generated by revolving a parametrized curve about the x or y axes.
2.3 Use of polar coordinates to analyze curves	2.3.1. Convert from rectangular to polar coordinates and vice versa. 2.3.2. Graph curves given in polar coordinates. 2.3.3. Find dy/dx on the graph of an equation in polar coordinates. 2.3.4. Find horizontal and vertical tangents to graphs in polar coordinates. 2.3.5. Find tangents at the pole. 2.3.6. Find the points of intersection of graphs in polar coordinates.
2.4 Applications of polar coordinates	2.4.1. Find the area of a region in polar coordinates. 2.4.2. Find the arc length of a curve in polar coordinates. 2.4.3. Find the surface area of a surface of revolution generated by revolving a curve given in polar coordinates.
3. *Vector valued functions*	
3.1 Investigation of vectors in the plane and in space	3.1.1. Perform vector addition and scalar multiplication. 3.1.2. Recognize the axioms of a vector space. 3.1.3. Find the length and direction of a vector. 3.1.4. Be conversant with two different vector notations: row vectors and the i,j,k notation. 3.1.5. Compute the dot product of two vectors. 3.1.6. Give a geometric interpretation of the dot product. 3.1.7. State the properties of the dot product. 3.1.8. Find the angle between two vectors. 3.1.9. Determine when two vectors are orthogonal. 3.1.10. Compute the direction cosines of a vector. 3.1.11. Find the orthogonal projection of one vector onto another. 3.1.12. Write a vector as an orthogonal sum of two vectors. 3.1.13. Compute the cross product of two vectors. 3.1.14. Give a geometric interpretation of the cross product. 3.1.15. State the properties of the cross product. 3.1.16. Give a geometric interpretation of the triple scalar product.
3.2 Description of lines and planes in space	3.2.1. Write the parametric equations of a line. 3.2.2. Write the symmetric equations of a line. 3.2.3. Write the equation of a plane. 3.2.4. Find the angle between two intersecting planes or lines.3 3.2.5. Find the line common to two intersecting planes. 3.2.6. Find the distance between a point and a plane. 3.2.7. Find the distance between a point and a line.

OBJECTIVES	STANDARDS
Specific Performance Criteria	**Intermediate Learning Objectives**
3. *Vector valued functions (continued)*	
3.3 Study of vector calculus	3.3.1. Find the domain of a vector-valued function. 3.3.2. Sketch the graph of a vector-valued function as a curve in $\mathbb{R}^2$ and $\mathbb{R}^3$. 3.3.3. Define the derivative of a vector-valued function. 3.3.4. Differentiate and integrate vector-valued functions. 3.3.5. Apply the properties of the derivative: linearity, product rules, and the chain rule.
3.4 Applications of vector calculus	3.4.1. Define velocity, acceleration, and speed. 3.4.2. Find the unit tangent vector and the principal unit normal vector to a curve. 3.4.3. Find the tangential and normal components of acceleration. 3.4.4. Find the arc length of a space curve given by a vector-valued function. 3.4.5. Use the arc length parameter to parametrize the curve. 3.4.6. Define curvature. 3.4.7. Use various formulas for curvature.
4. *Three dimensional drawings*	
4.1 Analyzing surfaces in space	4.1.1. Recognize when an equation describes a cylindrical surface. 4.1.2. Recognize the shape of a quadric surface from a second-degree equation. 4.1.3. Write the equation for a surface of revolution. 4.1.4. Transform coordinates between rectangular, cylindrical, and spherical coordinate systems. 4.1.5. Sketch surfaces in cylindrical and spherical coordinates.
5. *Multivariable calculus*	
5.1 Study of functions of more than one variable	5.1.1. Find the domain of a function of two or more variables. 5.1.2. Sketch a function of two variables as a surface. 5.1.3. Describe the level curves of a function of two variables. 5.1.4. Describe the level surfaces of a function of three variables. 5.1.5. Investigate limits and continuity for functions of two variables (intuitively). 5.1.6. Define Partial Derivatives. 5.1.7. Find the first and second partial derivatives. 5.1.8. Find the total differential. 5.1.9. Apply the appropriate chain rule in differentiation. 5.1.10. Find partial derivatives of an implicitly-defined function. 5.1.11. Find and interpret directional derivatives and the gradient. 5.1.12. State and prove the properties of the gradient. 5.1.13. Find the equation of the tangent plane and the normal line to a surface at a point.
6. *Optimization*	
6.1 Solutions of optimization problems	6.1.1. Determine critical points. 6.1.2. Find relative extrema for functions of two variables. 6.1.3. Apply the second partials test to determine extrema. 6.1.4. Use the method of Lagrange Multipliers to determine extrema.
7. *Double and triple integrals*	
7.1 Evaluating double and triple integrals	7.1.1. Evaluate iterated integrals. 7.1.2. Define the double and triple integrals as Riemann sums. 7.1.3. State the properties of the double and triple integrals. 7.1.4. Apply Fubini's Theorem in rectangular coordinates. 7.1.5. Transform a double integral from rectangular to polar coordinates. 7.1.6. Transform a triple integral from rectangular to cylindrical and spherical coordinates. 7.1.7. Define the Jacobian. 7.1.8. Use the Jacobian to change variables in evaluating a double integral.
7.2 Applications of the double and triple integrals	7.2.1. Find areas, volumes, mass, centre of mass and surface area using the double integral. 7.2.2. Find volumes using the triple integral.
8. *Integration, comprehensive assessment and exit profile goals*	
8.1 Recognition of the links between science, technology and the evolution of society.	8.1.1. Discuss the application of Calculus III to a relevant problem from science or engineering.
8.2 Development of a personal system of values.	8.2.1. Discuss any social or ethical aspects of the specific problem used for your comprehensive assessment.
8.3 Application of acquired knowledge to a new situation.	8.3.1. Demonstrate clearly the specific mathematical techniques used in solving some problem from science or engineering.
8.4 Clear demonstration of links between mathematics and at least one other science discipline	8.4.1. Apply knowledge or skills that have been acquired in Calculus III to topic(s) in physics, chemistry, biology, or engineering.

Methodology. This course will be 75 hours, meeting three times a week for a total of five hours. The main technique used is the lecture approach. Other methods that may be used are: problem-solving sessions, class discussions, assigned reading for independent study, and computer lab projects. Regular homework involving a minimum of five hours per week should be expected. Students are responsible for all problems and exercises in the text relevant to material covered in class.

The Mathematics Lab (H-022) functions both as a study area and as a centre where students may seek help with their mathematics courses. There are several computers equipped with the Maple software program available for student use.

Other Resources.

Math Website.
http://departments.johnabbott.qc.ca/departments/mathematics

Math Lab. Located in H-022; open from 9:00 to 16:00 (weekdays) as a study area, and from 11:30 to 16:00 for borrowing course materials or using the computers and printers for math assignments.

Math Help Centre. Located in H-022; teachers are on duty from 9:00 until 16:00 to give math help on a drop-in basis.

Academic Success Centre. The Academic Success Centre, located in H-117, offers study skills workshops and individual tutoring.

Departmental Attendance Policy. Regular attendance is expected. Missing six classes is grounds for automatic failure in this course. Many of the failures in this course are due to students missing classes.

Evaluation. A student's Final Grade is a combination of the Class Mark and the mark on the Final Exam. The Class mark will include three or four tests worth 75%, and other homework and assignments will make up the balance.

The Final Exam is set by the Course Committee (which consists of the instructors currently teaching this course).

Students must be available until the end of the final examination period to write exams.

The Final Grade will be the better of:

- 50% Class Mark and 50% Final Exam Mark, OR
- 25% Class Mark and 75% Final Exam Mark

Students choosing not to write the final examination will receive a failing grade of 50% or their class mark, whichever is less.

College Policies. Article numbers refer to the IPESA (Institutional Policy on the Evaluation of Student Achievement, available at http://johnabbott.qc.ca/ipesa). Students are encouraged to consult the IPESA to learn more about their rights and responsibilities.

Changes to Evaluation Plan in Course Outline (Article 4.3). Changes to the evaluation plan, during the semester, require unanimous consent.

Mid-Semester Assessment MSA (Article 3.3). Students will receive an MSA in accordance with College procedures.

Religious Holidays (Article 3.2). Students who wish to observe religious holidays must inform their teacher in writing within the first two weeks of the semester of their intent.

Grade Reviews (Article 3.2, item 19). It is the responsibility of students to keep all assessed material returned to them in the event of a grade review. (The deadline for a Grade Review is 4 weeks after the start of the next regular semester.)

Results of Evaluations (Article 3.3, item 7). Students have the right to receive the results of evaluation, for regular day division courses, within two weeks. For evaluations at the end of the semester/course, the results must be given to the student by the grade submission deadline.

Cheating and Plagiarism (Articles 8.1 & 8.2). Cheating and plagiarism are serious infractions against academic integrity, which is highly valued at the College; they are unacceptable at John Abbott College. Students are expected to conduct themselves accordingly and must be responsible for all of their actions.

Course Content (with selected exercises). The exercises listed below (referring to the textbook) should help you practice and learn the material taught in this course; they form a good basis for homework. Your teacher may supplement this list during the semester. Regular work done as the course progresses should make it easier for you to master the course.

Chapter 8: Infinite Series.

8.5 Power Series (1-20, 29, 30)
8.6 Representations of Functions as Power Series (1-11, 13-18, 23-30, 37, 38)
8.7 Taylor and Maclaurin Series (23-36, 39-64)
8.8 Applications of Taylor Polynomials (9ab, 10ab, 15ab, 16ab)

Chapter 9: Parametric Equations & Polar Coordinates.

9.1 Plane Curves Defined by Parametric Equations (1-18, 28)
9.2 Calculus with Parametric Curves (1-12, 21, 27, 29, 30, 31, 33-36, 47)
9.3 Polar Coordinates (1-20, 23-36, 43, 49, 51-54)
9.4 Areas and Lengths in Polar Coordinates (1, 5-12, 15-27, 29-31, 33, 34)

Chapter 10: Vectors and the Geometry of Space.

10.1-10.5 Vector Geometry (Review)
10.6 Cylinders and Quadric Surfaces (1, 3-9, 11, 13, 15, 17, 21-30)
10.7 Vector Functions and Space Curves (1-12, 17-23, 33-53, 57-64, 75-77)
10.8 Arclength and Curvature (1-4, 7-8,11-19, 33-38, 44)
10.9 Velocity and Acceleration (1-12, 18, 28-31)

Chapter 11: Partial Derivatives.

11.1 Functions of Several Variables (5-20 ,25-34, 47-50)
11.2 Limits and Continuity (3-15, 22-26, 29, 30)
11.3 Partial Derivatives (7-32, 37-40, 43-55, 58, 60, 61)
11.4 Tangent Planes and Linear Approximations (1, 3, 11, 13, 15, 17, 18-28)
11.5 The Chain Rule (1-8, 11, 13, 17-28, 32, 37, 39, 41, 45)
11.6 Directional Derivatives and the Gradient Vector (1-9, 15-17, 25, 31-33)
11.7 Maximum and Minimum Values (1, 3, 5, 7, 9, 13, 23-27, 31, 35, 39, 43)
11.8 Lagrange Multipliers (1-13, 17)

Chapter 12: Multiple Integrals.

12.1 Double Integrals Over a Rectangular Region (11-27, 29, 33)
12.2 Double Integrals Over General Regions (1-25, 31-42, 44)
12.3 Double Integrals in Polar Coordinates (1-4, 6-16, 21-26, 29)
12.5 Triple Integrals (3-6, 11, 15, 17-20, 21a, 25-27, 31, 33)
12.6 Triple Integrals in Cylindrical Coordinates (1-12, 15-19, 22, 27-28)
12.7 Triple Integrals in Spherical Coordinates (1-15, 17, 18, 21, 23, 26, 27, 35, 36)
12.8 Change in Variables in Multiple Integration (1-6, 11, 13, 15, 21, 23)