JOHN ABBOTT CEGEP/COLLEGE

Linear Algebra

General Information.
Ponderation: 2-2-2 *Credits*: 2 *Prerequisite*: 201-AS1-AB or equivalent
Objectives:

- O0UQ: To apply the methods of linear algebra and vector geometry to problem solving.
- 00UU: To apply acquired knowledge to one or more subjects in the science.

This course outline applies to all sections of 201-AS4-AB. Your instructor will include an appendix giving specific details of their schedule and availability.

Students are strongly advised to seek help from their instructor as soon as they encounter difficulties in the course.

Introduction. Linear Algebra I is the fourth Mathematics course in the Arts & Science Program. It is generally taken in the fourth semester. Linear Algebra, a useful tool in Physics, Chemistry and Computer Science, introduces the student to matrices, vector spaces and vector geometry.

The primary purpose of the course is the attainment of Objective 00UQ (To apply the methods of linear algebra and vector geometry to problem solving). To achieve this goal, the course must help the student understand the following basic concepts: systems of linear equations, matrices, determinants, two-dimensional and three-dimensional vectors from both an algebraic and a geometric perspective, applications of two-dimensional, three-dimensional, and n-dimensional vectors. The course will emphasize the ways in which geometric and algebraic concepts reinforce each other, and the ways in which these concepts can be generalized and applied to practical situations.

Emphasis is also placed on clarity and rigour in reasoning and in the application of methods. In addition to understanding and applying the basic concepts, the student will learn to prove simple propositions. This is perhaps the first course in which the student is required to produce a logical argument expressed in words, rather than just a numerical solution. The basic concepts are illustrated by applying them to various problems, which may be numerical, theoretical or applied in nature, where their application helps arrive at a solution. In this way, the course encourages the student to apply learning in one context to problems arising in another.

All students have access to the Mathematics Lab where suitable mathematical software programs, including MAPLE, are available for student use. The course uses a standard college, introductory level Linear Algebra textbook, and prepares the student for 201-DDC-05 (Linear Algebra II).

OBJECTIVES	STANDARDS
Statement of the competency To apply the methods of linear algebra and vector geometry to problem solving (00UQ)	**General Performance Criteria** • Appropriate use of concepts • Accurate representation of situations in the form of vectors and matrices • correct application of algorithms • Correct representations of loci • Proper justification of steps in the solution • Correct algebraic operations in conformity with rules • Appropriate use of terminology
Elements of the Competency 1. To solve systems of linear equations using matrices. 2. To express concrete problems as linear equations and solve the resulting system using matrices. 3. To develop matrix theory and to demonstrate related propositions. 4. To evaluate the determinant of a square matrix and to examine the relationship between the determinant and the inverse of a matrix. 5. To examine two-dimensional and three-dimensional vector spaces from both an algebraic and a geometric perspective. 6. To determine mathematical representations for lines and planes, to sketch lines in $\Re^2$, and lines and planes in $\Re^3$, and to determine intersections involving lines and planes. 7. To use vectors to calculate areas, volumes and distances. 8. To develop theoretical concepts pertaining to a vector space, with specific reference to $\Re^n$.	**Specific Performance Criteria** *[Specific performance criteria for each of these elements of the competency are shown below with the corresponding intermediate learning objectives. For the items in the list of learning objectives, it is understood that each is preceded by: "The student is expected to . . . "]*

Specific Performance Criteria	Intermediate Learning Objectives
1. *Systems of linear equations*	
1.1 Using augmented matrix and row operations to solve linear systems.	1.1.1. Write the augmented matrix of a linear system. 1.1.2. Define elementary row operations. 1.1.3. Solve linear systems using Gaussian elimination and Gauss-Jordan elimination (consistent and inconsistent systems). 1.1.4. Determine consistency conditions related to the solutions of linear systems.
2. *Applications of linear systems*	
2.1 Using linear systems to solve applied problems.	2.1.1. Set up and solve a system of linear equations in a variety of related problems. For example, • find interpolating polynomials; • Use Kirchoff's Law and Ohm's Law to set up and solve a linear system to determine the currents in simple electrical circuits; • set up and solve a system of equations to balance chemical equations.
3. *Matrices*	
3.1 Performing operations on matrices.	3.1.1. Give the definition of a matrix. 3.1.2. Determine whether two matrices are equal. 3.1.3. Define matrix operations (addition, subtraction, scalar multiplication, matrix multiplication, transpose of a matrix). 3.1.4. State, illustrate and prove properties of these operations. 3.1.5. Calculate a matrix that is the result of a series of matrix operations. 3.1.6. Identify a square matrix, a zero matrix, an identity matrix, the inverse of a matrix, a triangular matrix, a diagonal matrix, a symmetric matrix, a skew-symmetric matrix.
3.2 Performing operations on matrices involving the inverse of a matrix.	3.2.1. Find the inverse of a matrix by row reduction. 3.2.2. State, illustrate and prove inverse properties. 3.2.3. Use the inverse of a matrix and properties of the transpose to prove identities or solve matrix equations. 3.2.4. Write a system of equations as a matrix equation. 3.2.5. Interpret a matrix product as a linear transformation. 3.2.6. Solve a linear system by finding the inverse of its matrix of coefficients (if possible). 3.2.7. State, illustrate and prove statements characterizing invertible (or nonsingular) square matrices.
3.3 Performing operations on matrices involving elementary matrices.	3.3.1. Define elementary matrices. 3.3.2. Write a matrix and its inverse as a product of elementary matrices. 3.3.3. Relate row equivalence and elementary matrices. 3.3.4. Use elementary matrices to determine an LU factorization of a matrix A, and use this factorization to solve linear systems with coefficient matrix A.
4. *Determinants and inverses*	
4.1 Evaluate determinants using cofactor expansion and properties of the determinant.	4.1.1. Define the determinant of a 2×2 matrix. 4.1.2. State and illustrate properties of the determinant. 4.1.3. Calculate the determinant of a matrix by cofactor expansion. 4.1.4. Calculate the determinant of a matrix using a combination of row (or column) operations and cofactor expansion.
4.2 Prove simple propositions involving determinants.	4.2.1. Extend 3.2.6 to include the relationship between the value of the determinant and the existence of the inverse of a matrix. 4.2.2. Use properties of the determinant to prove identities or simple propositions.
4.3 Use of determinants to find the inverse of a matrix and to solve systems of linear equations.	4.3.1. Use the determinant of a matrix to determine whether a matrix is invertible. 4.3.2. Find the adjoint of a matrix. 4.3.3. Find the inverse of a matrix using the adjoint and determinant. 4.3.4. Solve linear systems using Cramer's rule.
5. *Vectors*	
5.1 Defining a vector algebraically and geometrically.	5.1.1. State the algebraic and geometric definition of a vector in $\mathbb{R}^2$. 5.1.2. State the algebraic and geometric definition of a vector in $\mathbb{R}^3$. 5.1.3. Define the equality of two vectors algebraically and geometrically.
5.2 Performing operations on vectors.	5.2.1. Define vector operations (addition, subtraction and scalar multiplication) both algebraically and geometrically. 5.2.2. State, illustrate and prove properties of these operations both algebraically and geometrically. 5.2.3. Calculate a vector that is the result of a series of vector operations both algebraically and geometrically. 5.2.4. Recognize standard notations for vectors. 5.2.5. Find the magnitude of a vector. 5.2.6. find a unit vector with the same direction (and sense) as a given vector. 5.2.7. Determine whether or not two vectors are parallel.
5.3 Calculation and interpretation of the dot product of two vectors.	5.3.1. Define the dot product of two vectors. 5.3.2. State, illustrate and prove properties of the dot product of two vectors. 5.3.3. Use the dot product to find the angle between two vectors. In particular, use the dot product to determine whether two vectors are perpendicular. 5.3.4. Use the dot product to find the projection of one vector onto another vector.

Specific Performance Criteria	Intermediate Learning Objectives
5.4 Calculation and interpretation of the cross product of two vectors.	5.4.1. Define the cross product of two vectors. 5.4.2. Give a geometric interpretation of the cross product of two vectors. 5.4.3. State, illustrate and prove properties of the cross product of two vectors. 5.4.4. Interpret a cross product equal to **0**
6. *Lines and Planes*	
6.1 Determination of equations of lines and planes.	6.1.1. Find the equation of a line in $\mathbb{R}^3$ in (i) standard form, (ii) parametric form and (iii) vector form and be able to convert from one form to the other. 6.1.2. Given one point on a line and the vector direction of that line, find the equation of a line in $\mathbb{R}^3$ in (i) parametric and (ii) vector form and be able to convert between these forms. 6.1.3. Find the equation of a line in $\mathbb{R}^3$ given various conditions such as (i) the line contains two point, (ii) the line contains a point and is parallel to a given line (iii) the line contains a point and is perpendicular to a given plane (iv) the line contains a point and is parallel to two given planes. 6.1.4. given one point on a plane and the normal of that plane, find the equation of a plane in (i) standard form, (ii) vector form and (iii) parametric form and be able to convert from one form to the other. 6.1.5. Find the equation of a plane in $\mathbb{R}^3$ given various conditions such as (i) the plane contains a point and the normal is given, (ii) the plane contains three noncollinear points, (iii) the plane contains a point and is perpendicular to two given planes, (iv) the plane contains a point and is perpendicular to a given line, (v) the plane contains two lines and (vi) the plane contains a point and a line.
6.2 Use of Cartesian axis systems to sketch lines and planes.	6.2.1. Develop the Cartesian axis systems for $\mathbb{R}^3$. 6.2.2. Plot points and vectors in a Cartesian axis system for $\mathbb{R}^3$. 6.2.3. Sketch lines in $\mathbb{R}^2$ and in $\mathbb{R}^3$. 6.2.4. Sketch planes in $\mathbb{R}^3$.
6.3 Determination of the intersection of a line and a plane, the intersection of two lines and the intersection of two or more planes.	6.3.1. Determine the intersection of a line and a plane. The line may lie on the plane (infinite intersection), or the line may be parallel to the plane (no intersection) or the line may intersect the plane in a single point (unique intersection). 6.3.2. Determine whether two lines intersect in a single point or are identical lines or are parallel but non-identical or are skew. 6.3.3. Use an augmented matrix to determine the intersection of two or more planes.
7. *Areas, volumes and distances*	
7.1 Calculation of areas and volumes using vector products.	7.1.1. Use the cross product to find the area of a parallelogram and the area of a triangle. 7.1.2. Define the scalar triple product and use this product to find the volume of a parallelepiped. 7.1.3. Interpret a scalar triple product of 0 (*i.e.*., three vectors are linearly dependent).
7.2 Calculation of distances using the dot product and cross products.	7.2.1. Calculate the distance from a point to a plane, or the distance between parallel planes. 7.2.2. Calculate the distance from a point to a line, the the distance between parallel planes. 7.2.3. Calculate the distance between skew lines.
8. *Vector spaces*	
8.1 Determination of a vector space.	8.1.1. State the definition of a vector space, its operations and its properties. 8.1.2. Verify that $\mathbb{R}^n$ satisfies the criteria in 8.1.1 for every positive integer n 8.1.3. Define a subspace of vector space and determine whether or not a given subset of a vector space is a subspace.
8.2 Use of the concepts of linear combinations, linear independence (dependence) and spanning.	8.2.1. State the definition of a linear combination of vectors and determine whether a given vector is a linear combination of some other vectors. 8.2.2. State the definition of linear independence and dependence and determine whether an indexed set of vectors is linearly independent or linearly dependent. 8.2.3. State the definition of the span of a set of vectors and determine the span of a set of vectors. 8.2.4. Interpret geometrically linear combinations, linear independence (dependence) and spanning.
8.3 Determination of a basis for a vector space.	8.3.1. State the definition of a basis for a vector space or a subspace and determine a basis for a given vector space or subspace. 8.3.2. Determine the dimension of a given vector space or subspace.
8.4 Determination of the null space, column space and row space of a matrix A.	8.4.1. Define the null space, column space and row space of a matrix A. Find a basis for, and the dimension of, each of these spaces.
8.5 Proofs of simple propositions.	8.5.1. Prove simple propositions involving the concepts of linear combinations, spanning, linear independence (dependence) and bases.

Course Content. Systems of linear equations in several variables, Gaussian elimination and row reduction, (column) vectors, vector equations, the matrix-vector product and matrix equations, solution sets of (homogeneous and nonhomogeneous) linear systems, linear (in)dependence, linear transformations, the standard matrix of a linear transformation, some applications of linear systems (including interpolating polynomials, balancing chemical equations and finding currents in electrical circuits), matrix operations, the inverse of a matrix, basic results on non-singular matrices, partitioned matrices, (P)LU factorization, subspaces (of spaces column vectors), the dimension of a subspace, the rank of a matrix and the rank equation, determinants as oriented volumes, properties of determinants and formulæ involving determinants, Cramer's rule, abstract vector spaces, subspaces of abstract vector spaces, the kernel and range of a linear transformation, the null space and the column space of a matrix, bases and coordinates, dimension and rank, distance, inner products, the Cauchy-Schwarz inequality, inner products, angles and orthogonality, projections, cross products and determinants, affine subspaces of vector spaces (or, "flats"), and their parametric and Cartesian representations, geometric examples in Euclidean spaces.

Required Texts. The textbook for this course is *Linear Algebra and its Applications (Custom Edition)*, by David C. Lay. The cost of the textbook is approximately about $117. The latest version of the text includes a supplementary reference on vector geometry called *Vector Geometry*, by Denis Sevee. For those students who do not purchase the latest version of the book, these supplementary notes are available for free at

http://tinyurl.com/VectorGeometryNotes

Teaching Methods. This course will be 60 hours, and meets in a classroom three times per week for a total of four hours per week. This course relies mainly on the lecture method, although at least one of the following techniques is used as well: question-and-answer sessions, labs, problem-solving periods, class discussions, and assigned reading for independent study. Failure to keep pace with the lecture results in a cumulative inability to cope with the material. It is entirely the student's responsibility to complete suggested homework assignments as soon as possible following the lecture. This allows the student the maximum benefit from any discussion of the homework (which usually occurs in the following class). *No calculators will be permitted on tests or the final exam.*

Other Resources.

Math Website.
http://departments.johnabbott.qc.ca/departments/mathematics

Math Lab. Located in H-022; open from 9:00 to 16:00 (weekdays) as a study area, and from 11:30 to 16:00 for borrowing course materials or using the computers and printers for math assignments.

Math Help Centre. Located in H-022; teachers are on duty from 9:00 until 16:00 to give math help on a drop-in basis.

Academic Success Centre. The Academic Success Centre, located in H-117, offers study skills workshops and individual tutoring.

Mathematics Department Attendance Policy. Regular attendance is expected. Many failures are due to students missing classes. Missing six classes is grounds for automatic failure in this course.

Evaluation Plan. The student's Final Grade is a combination of the Class Mark and the mark on the Final Exam. The Class Mark will include the student's results from three or more tests (worth at least 75% of the Class Mark), and possibly homework, quizzes and other assignments. The specifics of the Class Mark will be given by your instructor during the first week of classes in an appendix to this outline. Every effort is made to ensure equivalence between the various sections of this course. The Final Exam is set by the Course Committee (which consists of all instructors currently teaching this course).

The Final Grade will be the better of:

50% Class Mark and 50% Final Exam Mark
or
25% Class Mark and 75% Final Exam Mark

A student with a Class Mark of less than 50% may choose not to write the Final Exam, in which case the Class Mark will be assigned as the Final Grade.

Students must be available until the end of the final examination period to write exams.

Course Costs. Apart from the required texts, there are no additional costs.

College Policies. Article numbers refer to the IPESA (Institutional Policy on the Evaluation of Student Achievement, available at http://johnabbott.qc.ca/ipesa). Students are encouraged to consult the IPESA to learn more about their rights and responsibilities.

Changes to Evaluation Plan in Course Outline (Article 4.3). Changes to the evaluation plan, during the semester, require unanimous consent.

Mid-Semester Assessment MSA (Article 3.3). Students will receive an MSA in accordance with College procedures.

Religious Holidays (Article 3.2). Students who wish to observe religious holidays must inform their teacher in writing within the first two weeks of the semester of their intent.

Grade Reviews (Article 3.2, item 19). It is the responsibility of students to keep all assessed material returned to them in the event of a grade review. (The deadline for a Grade Review is 4 weeks after the start of the next regular semester.)

Results of Evaluations (Article 3.3, item 7). Students have the right to receive the results of evaluation, for regular day division courses, within two weeks. For evaluations at the end of the semester/course, the results must be given to the student by the grade submission deadline.

Cheating and Plagiarism (Articles 8.1 & 8.2). Cheating and plagiarism are serious infractions against academic integrity, which is highly valued at the College; they are unacceptable at John Abbott College. Students are expected to conduct themselves accordingly and must be responsible for all of their actions.

Selected Exercises. The exercises listed below should help you practice and learn the material taught in this course; they form a good basis for homework but they don't set a limit on the type of question that may be asked. Your teacher may supplement this list during the semester. Regular work done as the course progresses should make it easier for you to master the course.

The following list shows section numbers from Lay's textbook followed by the suggested problems. *All* problems in the supplementary notes on vector geometry by Denis Sevee are recommended.

1.1: 1–32
1.2: 1–31, 33
1.3: 1–26, 29–34
1.4: 1–36
1.5: 1–40
1.6: 5–10
1.7: 1–40
1.8: 1–31
1.9: 1–36
1.10: 5–8
Ch1 (Suppl. exercises): 1–24
2.1: 1–34
2.2: 1–10, 12–22, 29–35, 37, 38
2.3: 1–8, 11–24, 27–40
2.4: 1–16, 18–25
2.5: 1–16, 19–26
2.8: 1–36
2.9 1–28
Ch2 (Suppl. exercises): 1–18
3.1: 1–42
3.2: 1–36, 39–44
3.3: 1–32
Ch3 (Suppl. exercises): 1–17
4.1: 1–34
4.2: 1–36
4.3: 1–27, 29–36
4.5: 1–32
4.6: 1–34