JOHN ABBOTT CEGEP/COLLEGE

# Integral Calculus

**General Information.**
*Discipline*: Mathematics | *Course code*: 201-AS2-AB
*Ponderation*: 2-2-2 | *Credits*: 2
*Prerequisite*: 201-AS1-AB (Differential Calculus)
*Objective*: To solve problems using integral calculus (01Y2)

> *Students are strongly advised to seek help promptly from their teacher if they encounter difficulties in the course.*

**Introduction.** Integral Calculus is the sequel to Differential Calculus, and so is the second mathematics course in the Arts and Sciences program; it is generally taken in the second semester. The student will already be familiar with the notions of definite and indefinite integration from Differential Calculus. In Integral Calculus, these notions are studied in greater depth and their use in other areas of science, such as physics and to a lesser extent chemistry, is explored. In addition, the course introduces the student to the concept of infinite series and to the representation of functions by power series.

The primary purpose of the course is the attainment of objective 01Y2 ("To solve problems using integral calculus"). To achieve this goal, the course must help the student understand the following basic concepts: indefinite and definite integrals, improper integrals, sequences, infinite series, and power series involving real-valued functions of one variable (including algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions).

Emphasis will be placed on clarity and rigour in reasoning and in the application of methods. The student will learn to use the techniques of integration in several contexts, and to interpret the integral both as an antiderivative and as a limit of a sum of products. The basic concepts are illustrated by applying them to various problems where their application helps arrive at a solution. In this way, the course encourages the student to apply learning acquired in one context to problems arising in another.

Calculators are not permitted during tests and final examinations. Students will have access to the Math Lab where suitable mathematical software, including MAPLE, is available for student use. The course uses a standard college level calculus textbook, chosen by the Calculus I and Calculus II course committees.

**Course Objectives.** See below.

**Required Text.** *Single Variable Calculus: Early Transcendentals, 8th edition*, by James Stewart (Brooks/Cole, Cengage Learning 2012). Available from the college bookstore for about $121.

**Course Content** (with selected exercises)**.** The exercises listed on the next page should help you practice and learn the material taught in this course; they form a good basis for homework but they don't set a limit on the type of question that may be asked. Your teacher may supplement this list or assign equivalent exercises during the semester. Regular work done as the course progresses should make it easier for you to master the course.

| OBJECTIVES | STANDARDS |
|---|---|
| **Statement of the Competency**<br><br>To solve problems using integral calculus (01Y2). | **General Performance Criteria**<br><br>• Representation of a situation as a function: clear description of the relevant variables and precise formulation of the function<br>• Adequate representation of surfaces and solids of revolution<br>• Appropriate use of concepts<br>• Appropriate use of terminology<br>• Correct choice and application of integration techniques<br>• Correct algebraic operations<br>• Accurate calculations<br>• Correct interpretation of results<br>• Proper justification of steps in a solution |
| **Elements of the Competency**<br><br>1. Represent the problem as a real-valued function of one variable.<br>2. Apply integral calculus to solve the problem.<br>3. Evaluate the results in terms of the problem to be solved.<br>4. Explain the problem-solving process. | **Specific Performance Criteria**<br><br>*[Specific performance criteria for each of these elements of the competency are shown below with the corresponding intermediate learning objectives. For the items in the list of learning objectives, it is understood that each is preceded by: "The student is expected to . . . ". ]* |

| Specific Performance Criteria | Intermediate Learning Objectives |
|---|---|
| 1. *Indefinite integrals* | |
| 1.1 Use of basic substitutions to determine simple indefinite integrals. | 1.1.1. Express Calculus I differentiation rules as antidifferentiation rules.<br>1.1.2. Use these antidifferentiation rules and appropriate substitutions to calculate indefinite integrals. |
| 1.2 Use of more advanced techniques to determine more complex indefinite integrals. | 1.2.1. Use identities to prepare indefinite integrals for solution by substitution.<br>1.2.2. Evaluate an indefinite integral using integration by parts.<br>1.2.3. Evaluate an indefinite integral using trigonometric identities.<br>1.2.4. Evaluate an indefinite integral by partial fractions.<br>1.2.5. Evaluate an indefinite integral by selecting an appropriate technique.<br>1.2.6. Evaluate an indefinite integral by using a combination of techniques. |
| 2. *Limits of indeterminate forms* | |
| 2.1 Use of l'Hôpital's rule to determine limits of indeterminate forms. | 2.1.1. State l'Hôpital's rule and the conditions under which it is valid.<br>2.1.2. Calculate limits of the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ using l'Hôpital's rule.<br>2.1.3. For the indeterminate forms $0 \cdot \infty$, $\infty - \infty$, $1^{\infty}$, $0^0$, $\infty^0$, use the appropriate transformation to determine the limit using l'Hôpital's rule. |
| 3. *Definite and improper integrals* | |
| 3.1 Use of the Fundamental Theorem of Calculus to evaluate a definite integral. | 3.1.1. Use the Fundamental Theorem of Calculus to calculate definite integrals. |
| 3.2 Use of limits to calculate improper integrals. | 3.2.1. Calculate an improper integral where at least one of the bounds is not a real number.<br>3.2.2. Calculate an improper integral where the integrand is discontinuous at one or more points in the interval of integration. |
| 4. *Differential equations* | |
| 4.1 Use the language of differential equations to express physical problems. | 4.1.1. Translate a physical problem into the language of differential equations. |
| 4.2 Use of antidifferentiation to obtain general solutions to simple differential equations. | 4.2.1. Express a simple differential equation in the language of integration, and obtain the general solution. |
| 4.3 Use of antidifferentiation to obtain particular solutions to simple initial value problems. | 4.3.1. Express a simple initial value problem in the language of integration, and obtain the particular solution. |
| 5. *Areas, volumes, and lengths* | |
| 5.1 Use of differentials to set up definite integrals. | 5.1.1. Analyze a quantity $A$ as a sum $\sum \Delta A$ over an interval $[\,a, b\,]$; approximate $\Delta A$ by a product $f(x)\,dx$; conclude that $A$ is the definite integral $\int_a^b f(x)\,dx$. |
| 5.2 Calculation of areas of planar regions. | 5.2.1. Use 5.1.1 to set up a definite integral to calculate an area.<br>5.2.2. Sketch the area between two functions ($y = f(x)$, $y = g(x)$) and use 5.2.1 to calculate the area.<br>5.2.3. Sketch the area between two functions ($x = f(y)$, $x = g(y)$) and use 5.2.1 to calculate the area.<br>5.2.4. Sketch the area between two curves and determine the most efficient way (5.2.2 or 5.2.3) to calculate the area. |
| 5.3 Calculation of volumes of revolution | 5.3.1. Sketch the three dimensional solid obtained by revolving a region (of type 5.2.2 or 5.2.3) around an axis.<br>5.3.2. Use 5.1.1 to set up a definite integral to calculate the volume of the solid (5.3.1) by cross-sections.<br>5.3.3. Use 5.1.1 to set up a definite integral to calculate the volume of the solid (5.3.1) by shells.<br>5.3.4. Determine the most efficient way (cross-sections or shells) to calculate the volume of the solid (5.3.1), and calculate the volume by that method. |
| 5.4 Calculation of lengths of curves. | 5.4.1. Use 5.1.1 to set up a definite integral to calculate the length of a curve.<br>5.4.2. Use 5.1.1 and 1.2 to calculate the length of a curve. |
| 6. *Infinite series* | |
| 6.1 Determination of the convergence or divergence of a sequence. | 6.1.1. State the definition of the limit of a sequence.<br>6.1.2. Determine whether a sequence converges, and calculate its limit if it does, using: properties of the limit of a sequence; l'Hôpital's Rule; the Squeeze Theorem; the convergence of bounded monotonic sequences. |
| 6.2 Determination of the convergence or divergence of an infinite series of positive terms. | 6.2.1. State the definition of convergence for an infinite series.<br>6.2.2. State the test for divergence of an infinite series.<br>6.2.3. Use 6.2.1 to determine if a telescoping series converges, and if so, calculate the sum.<br>6.2.4. State the criterion for the convergence of an infinite geometric series.<br>6.2.5. Calculate the sum of a convergent geometric series (6.2.4); use this to solve appropriate problems (*e.g.*, the distance travelled by a bouncing ball).<br>6.2.6. State the integral, $p$-series, (direct) comparison, limit comparison, ratio and ($n^{\text{th}}$) root tests for convergence of an infinite series.<br>6.2.7. Determine whether an infinite series converges or diverges by choosing (and using) correct methods among (6.2.1–6.2.6) |
| 6.3 Determination of the convergence, conditional or absolute, or divergence of an infinite series. | 6.3.1. State the definitions of absolute and conditional convergence of an infinite series.<br>6.3.2. State the definition of an alternating series.<br>6.3.3. State the criterion for the (conditional) convergence of an alternating series.<br>6.3.4. Determine if an infinite series is absolutely convergent, conditionally convergent, or divergent, using the methods of (6.2.1–6.2.7, 6.3.1–6.3.3). |
| 6.4 Expression of functions as power series | 6.4.1. Use the methods of (6.2, 6.3) to find the radius and interval of convergence for a power series.<br>6.4.2. State the definitions of the Taylor and Maclaurin polynomials of degree $n$ for a function $f$ centred at $a$.<br>6.4.3. State the definitions of the Taylor and Maclaurin series for a function $f$ centred at $a$.<br>6.4.4. Use 6.4.3 to approximate a function $f$ at a given point. |

An item beginning with a decimal number (e. g., 3.5) refers to a section in *Single Variable Calculus: Early Transcendentals*, 8th edition. Answers to odd-numbered exercises can be found in the back of the text. Solutions to *all* the exercises are posted in the Calculus 2 Community on the JAC Portal; also, the complete solutions manual can be consulted in the Math Lab (see below). Additional resources for the textbook may be found at

http://stewartcalculus.com/media/17_home.php

*Inverse trigonometric functions.*

1.6 Inverse Functions and Logarithms (63–72)
2.6 Limits at Infinity; Horizontal Asymptotes (35, 40)
3.5 Implicit Differentiation (49–57)
4.9 Antiderivatives (18, 22, 24, 33)
5.3 The Fundamental Theorem of Calculus (39, 42)
5.4 Indefinite Integrals and the Net Change Theorem (12, 41, 43)

*Techniques of Integration.*

5.5 The Substitution Rule (7–28, 30–35, 38–48, 53–71, 79, 87–91)
7.1 Integration by Parts (3–13, 15, 17–24, 26–42)
7.2 Trigonometric Integrals (1–31, 33–49)
7.3 Trigonometric Substitution (5–20, 22–30)
7.4 Integration of Rational Functions and Partial Fractions (1–36, 46–51, 53, 54)
7.5 Strategy for Integration (1–80; skip 53, 74)

*Improper Integrals.*

4.4 Indeterminate Forms and l'Hospital's Rule (13–67; skip 24, 28, 29, 38, 42, 58)
7.8 Improper Integrals (1, 2, 5–41, 58)

*Applications of Integration.*

6.1 Areas Between Curves (1–14, 17, 22, 23, 25, 27)
6.2 Volumes (1–12, 15–18)
6.3 Volumes by Cylindrical Shells (1–20, 21–25 part (a) only, 37, 38, 41–43)
8.1 Arc Length (9, 11, 14, 15, 17–20, 33)
9.3 Separable Equations (1–14, 16–20, 39, 42, 45–48)
9.4 Models for Population Growth (9, 11)

*Infinite Sequences and Series.*

11.1 Sequences (1–3, 13–18, 23–51)
11.2 Series (1–4, 17–47, 60–62; skip 45)
11.3 The Integral Test and Estimates of Sums (3–5, 21, 22, 29)
11.4 The Comparison Tests (1–31, 41, 44–46)
11.5 Alternating Series (2–7, 12–15)
11.6 Absolute Convergence and the Ratio and Root Tests (1–38)
11.7 Strategy for Testing Series (1–28, 30–34)
11.8 Power Series (3–21, 23–26, 29–31)
11.10 Taylor and Maclaurin Series (3–9, 11–14, 21–26)

**Teaching Methods.** This course will be 60 hours, meeting three times per week for a total of four hours per week. It relies mainly on the lecture method, although some of the following techniques may also be used: question-and-answer sessions, problem-solving periods, class discussions, and assigned reading for independent study. In general, each class begins with a question period on previous topics, then new material is introduced, followed by worked examples. No marks are deducted for absenteeism (however, see below). Failure to keep pace with the lectures results in a cumulative inability to cope with the material and a failure in the course. A student will generally succeed or fail depending on how many exercises have been attempted and solved successfully. It is entirely the student's responsibility to complete suggested homework assignments as soon as possible following the lecture. This allows the student the maximum benefit from any discussion of the homework (which usually occurs in the following class). Supplementary notes and problems may be provided as appropriate.

**Other Resources.**

*Math Website.*
http://departments.johnabbott.qc.ca/departments/mathematics

*Math Lab.* Located in H-022; open from 9:00 to 16:00 (weekdays) as a study area, and from 11:30 to 16:00 for borrowing course materials or using the computers and printers for math assignments.

*Math Help Centre.* Located in H-022; teachers are on duty from 9:00 until 16:00 to give math help on a drop-in basis.

*Academic Success Centre.* The Academic Success Centre, located in H-117, offers study skills workshops and individual tutoring.

**Departmental Attendance Policy.** Regular attendance is expected. Missing six classes is grounds for automatic failure in this course. Many of the failures in this course are due to students missing classes.

**Evaluation Plan.** The Final Grade is a combination of the Class Mark and the mark on the Final Exam. The Class Mark will include four tests (worth 80% of the Class Mark), and quizzes and assignments (worth 20% of the Class Mark).

The Final Grade will be the better of:

50% Class Mark and 50% Final Exam Mark
or
25% Class Mark and 75% Final Exam Mark

A student *choosing not to write* the Final Exam will receive a failing grade of 50% or their Class Mark, whichever is less.

**Students must be available until the end of the final examination period to write exams.**

**Course Costs.** None apart from the text listed above.

**College Policies.** Article numbers refer to the IPESA (Institutional Policy on the Evaluation of Student Achievement, available at http://johnabbott.qc.ca/ipesa). Students are encouraged to consult the IPESA to learn more about their rights and responsibilities.

*Changes to Evaluation Plan in Course Outline (Article 4.3).* Changes to the evaluation plan, during the semester, require unanimous consent.

*Mid-Semester Assessment MSA (Article 3.3).* Students will receive an MSA in accordance with College procedures.

*Religious Holidays (Article 3.2).* Students who wish to observe religious holidays must inform their teacher in writing within the first two weeks of the semester of their intent.

*Grade Reviews (Article 3.2, item 19).* It is the responsibility of students to keep all assessed material returned to them in the event of a grade review. (The deadline for a Grade Review is 4 weeks after the start of the next regular semester.)

*Results of Evaluations (Article 3.3, item 7).* Students have the right to receive the results of evaluation, for regular day division courses, within two weeks. For evaluations at the end of the semester/course, the results must be given to the student by the grade submission deadline.

*Cheating and Plagiarism (Articles 8.1 & 8.2).* Cheating and plagiarism are serious infractions against academic integrity, which is highly valued at the College; they are unacceptable at John Abbott College. Students are expected to conduct themselves accordingly and must be responsible for all of their actions.