JOHN ABBOTT CEGEP/COLLEGE

Differential Calculus

ARTS AND SCIENCES
COURSE OUTLINE
FALL 2016

General Information.
Discipline: Mathematics *Course code*: 201-AS1-EN
Ponderation: 3-2-3 *Credits*: $2\frac{2}{3}$
Prerequisite: Secondary V Mathematics (Technical & Scientific option or Science option) or equivalent
Objective: To solve problems using differential calculus (01Y1)

Your teacher will give you his/her schedule and availability.
Students are strongly advised to seek help promptly from their teacher if they encounter difficulties in the course.

Introduction. Differential Calculus is the first of the required mathematics courses in the Arts and Sciences program. It is usually taken in the first semester and it introduces the student to the limit processes that are so vital to the development of calculus. Since differential calculus is a basic tool in physics, some of the applications will be related to problems in physics. To a lesser extent, differential calculus can also be applied to problems in chemistry and biology.

The primary purpose of the course is the attainment of objective 01Y1 ("To solve problems using differential calculus"). To achieve this goal, the course must help the student understand the following basic concepts: limits, continuity and derivatives involving real-valued functions of one variable (including algebraic, trigonometric, exponential, and logarithmic functions).

Emphasis will be placed on clarity and rigour in reasoning and in the application of methods. The student will learn to interpret the derivative both as a mathematical tool and as a rate of change. The derivative will be used in various contexts including velocity, acceleration, curve sketching, optimization and related rates. The basic concepts are illustrated by applying them to various problems where their application helps arrive at a solution. In this way, the course encourages the student to apply learning acquired in one context to problems arising in another.

When appropriate, students will be encouraged to use a scientific calculator; however, no calculators are allowed during tests or the final examination. Students will have access to the Math Lab where suitable mathematical software, including MAPLE, is available for student use. The course uses a standard college level calculus textbook, chosen by the Calculus I and Calculus II course committees.

Course Objectives. See below.

Required Text. *Single Variable Calculus: Early Transcendentals, 8th edition*, by James Stewart (Brooks/Cole, Cengage Learning 2012). Available from the college bookstore for about $121.

Teaching Methods. This course will be 75 hours, meeting three times a week for a total of five hours a week. It relies mainly on the lecture method, although some of the following techniques are also used: question-and-answer sessions, labs, problem solving periods, class discussions, and assigned reading for independent study. In general, each class begins with a question period on previous topics, then new material is introduced, followed by worked examples. No marks are deducted for absenteeism (however, see below). Failure to keep pace with the lectures results in a cumulative inability to cope with the material and a failure in the course. A student will generally succeed or fail depending on how many problems have been attempted and solved successfully. It is entirely the student's responsibility to complete suggested homework assignments as soon as possible following the lecture, as the material will be fresher in his/her mind. This also allows the student the maximum benefit from any discussion of the homework (which usually occurs in the following class). Supplementary notes and problems may be provided as appropriate.

OBJECTIVES	STANDARDS
Statement of the Competency To solve problems using differential calculus (01Y1).	**General Performance Criteria** • Representation of a situation as a function: clear description of the relevant variables and precise formulation of the function • Accurate graphical representation of a function • Appropriate use of concepts • Appropriate use of terminology • Correct choice and application of the rules and techniques of differential calculus • Correct algebraic operations • Accurate calculations • Correct interpretation of results • Proper justification of steps in a solution
Elements of the Competency 1. Represent the problem as a real-valued function of one variable. 2. Apply differential calculus to solve the problem. 3. Evaluate the results in terms of the problem to be solved. 4. Explain the problem-solving process.	**Specific Performance Criteria** *[Specific performance criteria for each of these elements of the competency are shown below with the corresponding intermediate learning objectives. For the items in the list of learning objectives, it is understood that each is preceded by: "The student is expected to . . . ".]*

Specific Performance Criteria	**Intermediate Learning Objectives**
1. *Functions*	
1.1 Recognition of functions	1.1.1. Decide whether a given relation is a function from its graphical representation. 1.1.2. Recognize and name the following functions from their symbolic representations: $f(x) = c$ constant function $f(x) = ax + b$ linear function $f(x) = ax^2 + bx + c$ quadratic function $f(x) = \lvert x \rvert$ absolute value function $f(x) = \sqrt{x}$ square root function $f(x) = a^x$ exponential function $f(x) = \log_a x$ logarithmic function $f(x) = \sin x$ sine function $f(x) = \cos x$ cosine function $f(x) = \tan x$ tangent function $f(x) = \csc x$ cosecant function $f(x) = \sec x$ secant function $f(x) = \cot x$ cotangent function 1.1.3. Recognize and name the following function from its symbolic representation: $f(x) = \sqrt[n]{x}$ (nth root function). 1.1.4. Recognize and name the functions listed in 1.1.2 from their graphical representations.
1.2 Finding domain, range and intercepts	1.2.1. Find and state the domain of functions listed in 1.1.2 from both their graphical and their symbolic representations. 1.2.2. Find and state the range of functions listed in 1.1.2 and 1.1.3 from both their graphical and their symbolic representations. 1.2.3. Find and state the x- and y-intercepts, if they exist, of functions listed in 1.1.2 from both their graphical and their symbolic representations.
1.3 Graphing of functions	1.3.1. Graph the functions listed in 1.1.2. 1.3.2. Graph piecewise defined functions whose pieces are made up of the functions listed in 1.1.2. 1.3.3. Apply vertical and horizontal shifts and reflections about the horizontal and vertical axes, and any combination of these to the functions listed in 1.1.2.
1.4 Operations on functions	1.4.1. Perform addition, subtraction, multiplication, division and composition of functions. 1.4.2. Divide two polynomial functions and express the answer in the form $\frac{p(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$. 1.4.3. Find the value of a function at a point in its domain. 1.4.4. Evaluate $\frac{f(x+h) - f(x)}{h}$ (the difference quotient) for linear, quadratic and simple rational functions.
1.5 Appropriate use of functions to represent given situations	1.5.1. Given an applied problem, decide which function best represents the situation and express the relationship using appropriate notation.
2. *Limits, Continuity and Derivatives*	
2.1 Determination of Limits	2.1.1. Give an intuitive description of the limit of a function at a point. 2.1.2. Evaluate a limit of a function by viewing the graph of the function. 2.1.3. Estimate a limit numerically by using successive approximations (using a table of values). 2.1.4. Evaluate a limit analytically by direct substitution, factoring, rationalizing or simplifying rational expressions. 2.1.5. Evaluate analytically limits at infinity. 2.1.6. Evaluate one-sided limits. 2.1.7. Recognize and evaluate infinite limits.
2.2 Determination of whether a function is continuous at a point or on an interval	2.2.1. Define continuity of a function at a point; that is, state the three conditions which must be satisfied in order that a function be continuous at a point. 2.2.2. Use the definition of continuity to determine if a function is continuous at a specific point. 2.2.3. Determine on which interval(s) a function is continuous.

Specific Performance Criteria	Intermediate Learning Objectives
2.3 Use of the limit definition of the derivative	2.3.1. Define the derivative of a function as (i) the limit of a difference quotient, (ii) the slope of a tangent line, and (iii) the rate of change (in particular the velocity function associated with a position function). 2.3.2. Use the limit definition of the derivative to determine the derivative of a polynomial of degree 1, 2 or 3, square root and simple rational functions. 2.3.3. Use the limit definition of the derivative to determine the numerical value of the derivative at a given point. 2.3.4. Use the limit definition of the derivative to determine the slope of the tangent line to a curve at a specific point. 2.3.5. Use the limit definition of the derivative to determine the equation of the tangent line to a curve at a specific point.
2.4 Use of the graph of a function to determine whether a function is differentiable at a point or on an interval	2.4.1. Determine if the derivative of a function exists at a point or on an interval by examining the graph of the function.
3. *Rules and Techniques of Differentiation*	
3.1 Recognition of the equivalence of various derivative notations	3.1.1. Recognize different notations for the derivative of y with respect to x: y', $f'(x)$, $\frac{dy}{dx}$, $\frac{d}{dx}f(x)$, $D_x y$
3.2 Use of basic differentiation formulas and rules and proof of simple propositions	3.2.1. Recognize when and how to use the basic differentiation formulas: $\frac{d}{dx}[c] = 0$ $\quad$ $\frac{d}{dx}[\sin x] = \cos x$ $\frac{d}{dx}[x^n] = nx^{n-1}$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\frac{d}{dx}[e^x] = e^x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$ $\frac{d}{dx}[\ln x] = \frac{1}{x}$ $\quad$ $\frac{d}{dx}[\sec x] = \sec x \tan x$ $\frac{d}{dx}[a^x] = a^x \ln a$ $\quad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$ $\frac{d}{dx}[\log_a x] = \frac{1}{x \ln a}$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ 3.2.2. Recognize when and how to use the following differentiation formulas derived from the chain rule: $\frac{d}{dx}[u^n] = nu^{n-1}u'$ $\quad$ $\frac{d}{dx}[\sin u] = \cos u\, u'$ $\frac{d}{dx}[e^u] = e^u u'$ $\quad$ $\frac{d}{dx}[\cos u] = -\sin u\, u'$ $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ $\quad$ $\frac{d}{dx}[\tan u] = \sec^2 u\, u'$ $\frac{d}{dx}[a^u] = a^u(\ln a)u'$ $\quad$ $\frac{d}{dx}[\sec u] = \sec u \tan u\, u'$ $\frac{d}{dx}[\log_a u] = \frac{u'}{u \ln a}$ $\quad$ $\frac{d}{dx}[\csc u] = -\csc u \cot u\, u'$ $\frac{d}{dx}[\cot u] = -\csc^2 u\, u'$ 3.2.3. Recognize when and how to use the following rules: constant rule, power rule, constant multiple rule, sum and difference rule. 3.2.4. Recognize when and how to use the product, quotient and chain rules. 3.2.5. Prove a selection of the rules in 3.2.1 using the limit definition of the derivative.
3.3 Determination of whether a function is differentiable at a point or on an interval	3.3.1. Determine whether a function is differentiable at a specified point or on an interval using graphical, numerical, or analytical methods.
3.4 Use of differentiation rules to perform implicit and logarithmic differentiation	3.4.1. Recognize when and how to use implicit differentiation. 3.4.2. Recognize when and how to use logarithmic differentiation.
3.5 Evaluation and application of higher order derivatives	3.5.1. Find higher order derivatives. 3.5.2. Use higher-order derivatives to solve position, velocity and acceleration problems.
3.6 Use of derivatives to find the slope of a tangent (normal) line to a curve at a point	3.6.1. Use the differentiation rules listed in 3.2.1 and 3.2.2 to find the slope of the tangent line to a curve at a point. 3.6.2. Use the differentiation rules listed in 3.2.1 and 3.2.2 to find the equation of the tangent line to a curve at a point. 3.6.3. Use the differentiation rules listed in 3.2.1 and 3.2.2 to find the equation of the normal line to a curve at a point.

Specific Performance Criteria	**Intermediate Learning Objectives**
4. *Graphing of Functions*	
4.1 Use of the derivative and related concepts to analyze the variations of a function and to sketch a graph of the function	4.1.1. Find critical numbers. 4.1.2. Find intervals on which a function is increasing and decreasing using the sign of the first derivative. 4.1.3. Find relative and absolute extrema. 4.1.4. Use the first or second derivative test to decide whether the critical points represent relative maxima or relative minima. 4.1.5. Find inflection points. 4.1.6. Find intervals on which a function is concave up or concave down using the sign of the second derivative. 4.1.7. Use limits to find all vertical and horizontal asymptotes. 4.1.8. Use 4.1.1–4.1.7 to graph polynomial, rational, trigonometric, logarithmic and exponential functions.
4.2 Demonstration of the ability to understand abstract properties of continuous and differentiable functions, as illustrated by two simple standard theorems.	4.2.1. State the conditions necessary for Rolle's Theorem. 4.2.2. State the conclusion of Rolle's Theorem. 4.2.3. State the conditions necessary for the Mean Value Theorem. 4.2.4. State the conclusion of the Mean Value Theorem.
5. *Optimization and Rate-of-Change Problems*	
5.1 Solution of optimization problems	5.1.1. Represent an optimization word problem in functional form. 5.1.2. Determine the quantity, P, to be maximized or minimized and identify the variables which are involved. 5.1.3. Draw a diagram, if possible, to illustrate the problem and list any other relationship(s) between the variables. 5.1.4. Express P as a function of one variable. 5.1.5. Find the derivative of the function for P obtained in 5.1.4. 5.1.6. Find all the possible critical values by solving the equation $P' = 0$. 5.1.7. Test the critical value(s) and interval endpoints for absolute maximum or minimum. 5.1.8. Interpret (explain) the results found in the optimization problem.
5.2 Solution of problems involving related rates	5.2.1. Represent a word problem involving related reates in functional form. 5.2.2. Identify the variables and rates in the problem. 5.2.3. Draw a diagram, if possible, to illustrate the problem. 5.2.4. Determine the equation relating the variables. 5.2.5. Differentiate the equation in 5.2.4 with respect to time, t. 5.2.6. Solve the equation in 5.2.5 for the required rate. 5.2.7. Interpret (explain) the results found in the problem involving related rates.
6. *Integration*	
6.1 Evaluation of the indefinite integral	6.1.1. Give the definition of the indefinite integral as an antiderivative. 6.1.2. Express the basic differentiation formulas listed in 3.2.1 as antidifferentiation formulas. 6.1.3. Recognize when and how to use the constant multiple rule and the sum and difference rule in the evaluation of integrals. 6.1.4. Use the antidifferentiation formulas from 6.1.2 and the rules in 6.1.3 to evaluate indefinite integrals.
6.2 Evaluation of the definite integral	6.2.1. State the definition of the definite integral. 6.2.2. State the Fundamental Theorem of Calculus. 6.2.3. Find the definite integral of functions described in 6.1. 6.2.4. Use the Fundamental Theorem of Calculus to find the area of a region under a curve on a closed interval.
6.3 Solution of simple differential equations	6.3.1. Find the general solution to a differential equation of the form $y' = f(x)$. 6.3.2. Find the particular solution to a differential equation of the form $y' = f(x)$ given an initial condition $y(a) = b$.

Course Content (with selected exercises). This is a *minimal* list of exercises which you should attempt, assuming you are also doing regular homework (e.g., WEBWORK) assigned by your teacher.

1.1: 1, 2, 7, 9, 14, 49, 55, 63, 72, 73
1.2: 1, 3, 8, 10, 11, 17
1.3: 3, 6, 12, 24, 30, 31, 37, 41, 45, 53, 61, 63, 65
1.4: 12, 14, 21, 23, 30, 37
1.5: 18, 21, 37, 41, 50, 53, 56, 61
Chap. 1 Review (p. 69): 2, 10, 17, 18, 23, 24, 25a, 25b
2.1: [instructor's discretion]
2.2: 6, 9, 11, 17, 31, 35, 40
2.3: 2, 9, 15, 19, 23, 29, 37, 40, 43, 45, 53, 59, 64, 65
2.4: [instructor's discretion]
2.5: 4, 7, 21, 33, 35, 41, 46, 47, 52, 69, 71
2.6: 4, 9, 21, 24, 29, 32, 33, 39, 45, 49, 52, 54, 59, 63, 67
2.7: 3, 8, 10, 11, 17, 22, 23, 28, 35, 37, 39, 59, 60
2.8: 3, 25, 28, 30, 43, 51, 57, 63, 67
Chap. 2 Review (p. 166): 1, 2, 8, 12, 15, 17, 20, 22, 23, 29, 33, 36, 43, 47, 54
3.1: 3, 9, 23, 24, 27, 31, 37, 55, 60, 62, 66, 69, 81, 83, 86
3.2: 9, 15, 20, 23, 25, 26, 27, 32, 41, 44, 48, 49, 52, 53, 62
3.3: 5, 8, 13, 16, 20, 22, 32, 34, 37, 43, 45, 50, 52, 54, 58
3.4: 19, 25, 38, 40, 41, 44, 45, 48, 53, 59, 65, 74, 76, 78, 84
3.5: 7, 16, 19, 20, 21, 23, 29, 38, 39, 43, 46, 75, 77, 79, 80
3.6: 9, 12, 13, 21, 22, 29, 34, 40, 42, 45, 49, 50, 52, 56
3.7: 1, 5, 10
3.8: [instructor's discretion]
3.9: 12, 18, 23, 27, 29, 30, 33, 42, 44, 47, 50
Chap. 3 Review (p. 267): 28, 37, 41, 50, 53, 59, 60, 66, 81, 85, 89, 95, 98, 109, 112
4.1: 5, 7, 10, 13, 27, 39, 43, 44, 51, 56, 57, 63, 67, 72, 77
4.2: 9, 12, 18, 19, 22, 24, 25, 37, 38
4.3: 8, 11, 12, 16, 18, 19, 27, 30, 35, 45, 47, 55, 60, 75, 79, 82
4.5: 7, 13, 15, 24, 27, 30, 34, 39, 40, 43, 46, 50
4.7: 13, 16, 22, 33, 34, 36, 39, 50, 58, 68, 71, 73, 74
4.9: 5, 12, 15, 16, 29, 37, 38, 47, 50, 53, 55, 61, 63, 69, 77
Chap. 4 Review (p. 359): 5, 6, 17, 24, 29, 32, 47, 54, 72, 74, 78, 79, 84, 85
5.1: 5, 7, 17, 21, 25, 27
5.2: 4, 7, 17, 25, 30, 33, 37, 49, 53
5.3: 3, 7, 18, 26, 29, 33, 37, 41, 43, 63, 65, 73, 75, 83
5.4: 1, 4, 9, 14, 15, 16, 18, 29, 31, 37, 38, 46, 49, 61, 71
Chap. 5 Review (p. 422): 2, 5, 7, 12, 25, 40, 49, 58, 62, 67, 72

To strengthen your skills with more practice, attempt any of the exercises in the sections above which not omitted explicitly below. While doing so, it is a good idea to focus on types of problems with which you struggle.

Practice exercises: Omissions

1.5: Omit 63–76
Chap. 1 Review: omit 25c, 25d, 26d
2.5: omit 29, 32, 60
2.6: omit 35, 40
Chap. 2 Review: omit 19
3.5: omit 17, 49–64
Chap. 3 Review: omit 6, 12, 17, 31, 38, 43, 45, 47, 48
4.1: omit 42, 62
4.2: omit 34, 35
4.3: omit 56, 64
4.5: omit 41, 42, 45, 48, 51–54, 71
4.9: omit 18, 19, 22, 24, 33, 44
Chap. 4 Review: omit 7–14, 31, 33, 34, 61–64 , 66, 68, 73, 81–83
5.3: omit 38, 39, 42, 62
5.4: omit 12, 13, 30, 40, 41, 43, 48
Chap. 5 Review: omit 8, 14, 17–24, 26–38, 41, 42, 44, 56, 63, 65, 66, 71

Other Resources.

Math Website.
http://departments.johnabbott.qc.ca/departments/mathematics

Math Lab. Located in H-203 and open from 9:00 to 16:00 (weekdays) as a study area, and from 11:30 to 16:00 for borrowing course materials or using the computers and printers for math assignments.

Math Help Centre. Located in H-200A; teachers are on duty from 9:00 until 16:00 to give math help on a drop-in basis.

Peer Tutoring. Starting on the fifth week of each semester, first year students can be paired with a fellow finishing student for a weekly appointment in the Math Lab. Ask your teacher for details.

Academic Success Centre. The Academic Success Centre, located in H-117, offers study skills workshops and individual tutoring.

Departmental Attendance Policy. Regular attendance is expected. Missing six classes is grounds for automatic failure in this course. Many of the failures in this course are due to students missing classes.

Evaluation Plan. The Final Grade is a combination of the Class Mark and the mark on the Final Exam. The Class Mark will include four tests (equally weighted, and worth a total of 80% of the Class Mark) and assignments and quizzes (worth 20% of the Class Mark). The four tests are tentatively scheduled for September 15, October 13, November 10 and December 5.

The Final Grade will be the better of:

50% Class Mark and 50% Final Exam Mark
or
25% Class Mark and 75% Final Exam Mark

A student *choosing not to write* the Final Exam will receive a failing grade of 50% or their Class Mark, whichever is less. *Students must be available until the end of the final examination period to write exams.*

Course Costs. In addition to the cost of the textbook (see above), a scientific calculator ($15–$25) might be useful. *No calculators are allowed during tests or the final exam.*

College Policies. Article numbers refer to the IPESA (Institutional Policy on the Evaluation of Student Achievement), which can be found at the college website. Students are encouraged to consult the IPESA to learn more about their rights and responsibilities.

Changes to Evaluation Plan in Course Outline (Article 4.3). Changes to the evaluation plan, during the semester, require unanimous consent of students.

Mid-Semester Assessment MSA (Article 3.3). Students will receive an MSA in accordance with College procedures.

Religious Holidays (Article 3.2). Students who wish to observe religious holidays must inform their teacher of their intent, in writing, within the first two weeks of the semester.

Grade Reviews (Article 3.2, item 19). It is the responsibility of students to keep all assessed material returned to them in the event of a grade review. (The deadline for a Grade Review is 4 weeks after the start of the next regular semester.)

Results of Evaluations (Article 3.3, item 7). Students have the right to receive the results of evaluation, for regular day division courses, within two weeks. For evaluations at the end of the semester/course, the results must be given to the student by the grade submission deadline.

Cheating and Plagiarism (Articles 8.1 & 8.2). Cheating and plagiarism are serious infractions against academic integrity, which is highly valued at the College; they are unacceptable at John Abbott College. Students are expected to conduct themselves accordingly and must be responsible for all of their actions.