JOHN ABBOTT CEGEP/COLLEGE

Calculus I

SOCIAL SCIENCE PROGRAM
COURSE OUTLINE
WINTER 2019

General Information.
Discipline: Mathematics *Course code*: 201-103-RE
Ponderation: 3-2-3 *Credits*: $2\frac{2}{3}$
Prerequisite: Secondary V Math TS5 or SN5 (or equivalent)
Objectives: 022X, 022N

Your teacher will give you his/her schedule and availability.
Students are strongly advised to seek help promptly from their teacher if they encounter difficulties in the course.

Introduction. Calculus I is the first of the required mathematics courses in the Social Science program. It is usually taken in the first semester and it introduces the student to the limit processes that are so vital to the development of calculus. Since differential calculus is a basic tool in business, economics, psychology and sociology, some of the applications will be related to problems in these fields.

The primary purpose of the course is the attainment of objectives 022X and 022N (see below). To achieve this goal, the course will help the student understand the following basic concepts: limits, continuity and derivatives involving real-valued functions of one variable (including algebraic, trigonometric, exponential, and logarithmic functions).

Emphasis will be placed on clarity and rigour in reasoning and in the application of methods. The student will learn to interpret the derivative both as a mathematical tool and as a rate of change. The derivative will be used in various Social Science contexts such as population growth, the spread of disease or rumour, financial mathematics, curve sketching, optimization and cost-benefit analysis (cost, revenue, and profit). The basic concepts are illustrated by applying them to various problems where their application helps arrive at a solution. In this way the course encourages the student to apply learning acquired in one context to problems arising in another.

Only calculators which have previously been inspected and approved via sticker by the instructor will be permitted for use on quizzes, tests or the final examination. The only calculators that will be approved begin with the model number **SHARP EL-531**. An acceptable calculator model is available for purchase at the bookstore.

Students will also have access to computers where suitable mathematical software, including MAPLE, is available for student use. The course uses a standard college level calculus textbook, chosen by the Calculus I and Calculus II course committees.

Required Text. *Applied Calculus for the Managerial, Life, and Social Sciences, 10th edition*, by Soo T. Tan. Available from the college bookstore for about $130.

Evaluation Plan. A student's Final Grade is a combination of the Class Mark and the mark on the Final Exam. The Class Mark will be 75% (three to five tests) and 25% at your teacher's discretion (more tests, quizzes or assignments). The specifics of the Class Mark are included in an appendix that is distributed to students along with this course outline. The Final Exam is set by the 103 course committee (which consists of all instructors currently teaching this course), and is marked by each individual instructor. Every effort is made to ensure equivalence between the various sections of the course.

The Final Grade will be the better of:

50% Class Mark and 50% Final Exam Mark
or
25% Class Mark and 75% Final Exam Mark

A student *choosing not to write* the Final Exam will receive a failing grade of 50% or their Class Mark, whichever is less.

Students must be available until the end of the final examination period to write exams.

Teaching Methods. This course will be 75 hours, meeting three times a week for a total of five hours a week. It relies mainly on the lecture method, although some of the following techniques are also used: question-and-answer sessions, labs, problem-solving periods, and class discussions. In general, each class begins with a question period on previous topics, then new material is introduced, followed by worked examples. No marks are deducted for absenteeism (however, see below). Failure to keep pace with the lectures results in a cumulative inability to cope with the material and a failure in the course. A student will generally succeed or fail depending on how many problems have been attempted and solved successfully. It is entirely the student's responsibility to complete suggested homework assignments as soon as possible following the lecture, as the material will be fresher in his/her mind. This also allows the student the maximum benefit from any discussion of the homework (which usually occurs in the following class). Answers to a selected number of problems can be found in the back of the text. Individual teachers may provide supplementary notes and problems as they see fit.

Other Resources.

Math Website.
http://departments.johnabbott.qc.ca/departments/mathematics

Math Study Area. Located in H-200A and H-200B; the common area is usually open from 8:30 to 17:30 on weekdays as a quiet study space. Computers and printers are available for math-related assignments. It is also possible to borrow course materials when the attendant is present.

Math Help Centre. Located near H-211; teachers are on duty from 9:00 until 16:00 to give math help on a drop-in basis.

Peer Tutoring. Starting on the fifth week of each semester, first year students can be paired with a fellow finishing student for a weekly appointment of tutoring. Ask your teacher for details.

Academic Success Centre. The Academic Success Centre, located in H-117, offers study skills workshops and individual tutoring.

Course Costs. The textbook costs about $130, and the approved model of scientific calculator costs about $24.

Departmental Attendance Policy. Regular attendance is expected. Missing six classes is grounds for automatic failure in this course. Many of the failures in this course are due to students missing classes.

Course Content (with suggested exercises). The exercises listed below should help you practice and learn the material taught in the course; they form a good basis for homework. Your teacher may supplement this list during the semester. Regular work done as the course progresses should make it easier for you to master the course.

Review topics (sections A and B below) may be taught as needed. The final exam will cover topics from Unit 1 - 4.

A: Algebraic Concepts

1.1 Pre-Calculus Review I
Exercises: #7 - 18, 23 - 32, 33 - 48, 75-96, 99-106

1.2 Pre-Calculus Review II
Exercises: #1, 3-6, 17, 18, 29 - 34, 38 - 40, 45 - 71, 79 - 84

1.3 The Cartesian Coordinate System
Concept Questions: #1
Exercises: #1 - 20

1.4 Straight Lines
Self-Check: #1 - 3
Concept Questions: #1 - 4
Exercises: #1 - 22, 25 - 62, 64 - 67

B: Functions

2.1 Functions and Their Graphs
Self-Check: #1 - 3
Concept Questions: #1a,b, 3, 4
Exercises: #1 - 20, 23 - 38, 51 - 58

2.2 The Algebra of Functions
Self-Check: #1
Concept Questions: #1 - 4
Exercises: #1 - 52

2.3 Functions and Mathematical Models
Exercises: #9 - 16

Unit 1: Limits and Continuity

2.4 Limits
Self-Check: #1 - 2
Concept Questions: #1 - 5
Exercises: #1 - 8, 11-15, 16 - 80, 83, 84, 87, 89 - 94
Supplement A

2.5 One-Sided Limits and Continuity
Self-Check: #1 - 2
Concept Questions: #1 - 4
Exercises: 1 - 55, 57 - 60, 73, 74, 85 - 89, 91 - 93
Supplement B

2.6 The Derivative
Self-Check: #1 - 2
Concept Questions: #1, 2, 4, 5
Exercises: #9 - 23, 24 - 26 (parts a and b only), 47-52

Unit 2: The Derivative

3.1 Basic Rules of Differentiation
Self-Check: #1 - 2
Concept Questions: #1 - 3
Exercises: #1 - 36, 41 - 49

3.2 The Product and Quotient Rules
Self-Check: #1 - 2
Concept Questions: #1, 2
Exercises: #1 - 32, 35 - 50

3.3 The Chain Rule
Self-Check: #1
Concept Questions: #1
Exercises: #1 - 48, 63 - 66
Supplement C

3.5 Higher Order Derivatives
Self-Check: #1 - 2
Concept Questions: #1
Exercises: #1 - 28, 42, 44, 46

3.6 Implicit Differentiation
Self-Check: #1 - 2
Exercises: #1 - 34

5.4 Differentiation of Exponential Functions
Self-Check: #1 - 2
Concept Questions: #1 - 2
Exercises: #1 - 34

5.5 Differentiation of Logarithmic Functions
Self-Check: #1 - 2
Concept Questions: #2
Exercises: #1 - 66, 69, 70

12.3 Derivatives of Trigonometric Functions
Self-Check: #1 - 2
Concept Questions: #1 - 4
Exercises: #1 - 36

Supplement D

Unit 3: Marginals and Price Elasticity of Demand

3.4 Marginal Functions in Economics
Self-Check: #1 - 2
Concept Questions: #1 - 3
Exercises: #1 - 17, 23 - 30, 34 - 40

3.6 Implicit Differentiation
Exercises: #46, 47

5.2 Logarithmic Functions
Exercises: #58

5.4 Differentiation of Exponential Functions
Exercises: #73

Supplement E

Unit 4: Graphing and Optimization

4.1 Applications of the First Derivative
Self-Check: #1, 2
Concept Questions: #1 - 5
Exercises: #1 - 45, 49 - 74, 78, 84
Page 376 #35, 36, 43, 44, 36
Supplement F

4.2 Applications of the Second Derivative
Self-Check: #1 - 3
Concept Questions: #1 - 3
Exercises: #1 - 14, 17 - 20, 25 - 76
Page 376 #38, 39, 40
Supplement G

4.3 Curve Sketching
Self-Check: #1 - 2
Concept Questions: #1 - 4
Exercises: #1 - 30, 33 - 60

4.4 Optimization I
Self-Check: #1 - 3
Concept Questions: #1 - 2
Exercises: #1 - 38, 40, 42, 44, 46, 47 - 57, 61

4.5 Optimization II
Self-Check: #1
Exercises: #1 - 13, 17, 19 - 22

College Policies.
Policy No. 7 - IPESA, Institutional Policy on the Evaluation of Student Achievement: http://johnabbott.qc.ca/ipesa.

Changes to Evaluation Plan in Course Outline (Article 5.3). Changes require documented unanimous consent from regularly attending students and approval by the department and the program dean.

Religious Holidays (Article 3.2.13 and 4.1.6). Students who wish to miss classes in order to observe religious holidays must inform their teacher of their intent in writing within the first two weeks of the semester.

Student Rights and Responsibilities: (Article 3.2.18). It is the responsibility of students to keep all assessed material returned to them and/or all digital work submitted to the teacher in the event of a grade review. (The deadline for a Grade Review is 4 weeks after the start of the next regular semester.)

Student Rights and Responsibilities: (Article 3.3.6). Students have the right to receive graded evaluations, for regular day division courses, within two weeks after the due date or exam/test date, except in extenuating circumstances. A maximum of three (3) weeks may apply in certain circumstances (ex. major essays) if approved by the department and stated on the course outline. For evaluations at the end of the semester/course, the results must be given to the student by the grade submission deadline (see current Academic Calendar). For intensive courses (i.e.: intersession, abridged courses) and AEC courses, timely feedback must be adjusted accordingly.

Academic Procedure: Academic Integrity, Cheating and Plagiarism (Article 9.1 and 9.2). Cheating and plagiarism are unacceptable at John Abbott College. They represent infractions against academic integrity. Students are expected to conduct themselves accordingly and must be responsible for all of their actions.

College definition of Cheating: Cheating means any dishonest or deceptive practice relative to examinations, tests, quizzes, lab assignments, research papers or other forms of evaluation tasks. Cheating includes, but is not restricted to, making use of or being in possession of unauthorized material or devices and/or obtaining or providing unauthorized assistance in writing examinations, papers or any other evaluation task and submitting the same work in more than one course without the teachers permission. It is incumbent upon the department through the teacher to ensure students are forewarned about unauthorized material, devices or practices that are not permitted.

College definition of Plagiarism: Plagiarism is a form of cheating. It includes copying or paraphrasing (expressing the ideas of someone else in ones own words), of another person's work or the use of another persons work or ideas without acknowledgement of its source. Plagiarism can be from any source including books, magazines, electronic or photographic media or another student's paper or work.

OBJECTIVES	STANDARDS
Statement of the competency To apply the methods of differential calculus to the study of functional models in the field of Social Science (022X). To identify the contribution of knowledge related to the social science disciplines to the understanding of the human phenomena (022N).	**General Performance Criteria** • Basic knowledge of the historical context of the development of differential calculus • Appropriate use of concepts • Satisfactory representation of a situation as a function • Satisfactory graph of a function. • Algebraic operations in conformity with rules • Correct selection and application of rules and techniques • Accuracy of calculations • Explanation of the steps in the procedure for solving optimization and rate of change problems. • Correct interpretation of results • Use of appropriate terminology.
Elements of the Competency 1. To place the development of differential calculus in historical context. 2. To recognize and describe the characteristics of algebraic, exponential, logarithmic and trigonometric functions expressed in symbolic or graphic form. 3. To analyze the behaviour of a function represented in symbolic or graphic form using an intuitive approach to the concept of limits. 4. To define the derivative of a function, give its interpretation and apply differentiation techniques. 5. To analyze the variations of a function using differential calculus. 6. To solve optimization and rate of change problems.	**Specific Performance Criteria** *[Specific performance criteria for each of these elements of the competency are shown below with the corresponding intermediate learning objectives. For the items in the list of learning objectives, it is understood that each is preceded by: "The student is expected to . . . ".]*

Specific Performance Criteria	**Intermediate Learning Objectives**
1. *The Development of Calculus*	
1.1 The history of Calculus	1.1.1. Recognize Newton and Leibniz as the founders of calculus. 1.1.2. Examine the historical context of differential calculus by an investigation of the slope of a tangent line, a problem posed by Newton and Leibniz and its relevance in today's society.
2. *Functions*	
2.1 Recognition of functions	2.1.1. Decide whether a given relation is a function from its graphical representation. 2.1.2. Recognize and name the following functions from their symbolic representations: $f(x) = c$ constant function $f(x) = ax + b$ linear function $f(x) = ax^2 + bx + c$ quadratic function $f(x) = \lvert x \rvert$ absolute value function $f(x) = \sqrt{x}$ square root function $f(x) = a^x$ exponential function $f(x) = \log_a x$ logarithmic function $f(x) = \sin x$ sine function $f(x) = \cos x$ cosine function $f(x) = \tan x$ tangent function $f(x) = \csc x$ cosecant function $f(x) = \sec x$ secant function $f(x) = \cot x$ cotangent function 2.1.3. Recognize and name the following function from its symbolic representation: $f(x) = \sqrt[n]{x}$ (nth root function). 2.1.4. Recognize and name the functions listed in 2.1.2 from their graphical representations.
2.2 Finding domain, range and intercepts	2.2.1. Find and state the domain of functions listed in 2.1.2 from both their graphical and their symbolic representations. 2.2.2. Find and state the range of functions listed in 2.1.2 from both their graphical and their symbolic representations. 2.2.3. Find and state the x- and y-intercepts, if they exist, of functions listed in 2.1.2 from both their graphical and their symbolic representations.
2.3 Graphing of functions	2.3.1. Graph the functions listed in 2.1.2. 2.3.2. Graph piecewise defined functions whose pieces are made up of the functions listed in 2.1.2. 2.3.3. Apply vertical and horizontal shifts and reflections about the horizontal and vertical axes, and any combination of these to the functions listed in 2.1.2.
2.4 Operations on functions	2.4.1. Perform addition, subtraction, multiplication, division and composition of functions. 2.4.2. Divide two polynomial functions and express the answer in the form $\frac{p(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$. 2.4.3. Find the value of a function at a point in its domain. 2.4.4. Evaluate $\frac{f(x+h) - f(x)}{h}$ (the difference quotient) for a polynomial of degree 1, 2 or 3, square root and simple rational functions.
2.5 Appropriate use of functions to represent given situations	2.5.1. Given an applied problem, decide which function best represents the situation and express the relationship using appropriate notation.
3. *Limits and Continuity*	
3.1 Determination of Limits	3.1.1. Give an intuitive description of the limit of a function at a point. 3.1.2. Evaluate a limit of a function by viewing the graph of the function. 3.1.3. Estimate a limit numerically by using successive approximations (using a table of values). 3.1.4. Evaluate a limit analytically by direct substitution, factoring, rationalizing or simplifying rational expressions. 3.1.5. Evaluate analytically limits at infinity. 3.1.6. Evaluate one-sided limits. 3.1.7. Recognize and evaluate infinite limits.
3.2 Determination of whether a function is continuous at a point or on an interval	3.2.1. Define continuity of a function at a point; that is, state the three conditions which must be satisfied in order that a function be continuous at a point. 3.2.2. Use the definition of continuity to determine if a function is continuous at a specific point. 3.2.3. Determine on which interval(s) a function is continuous.
4. *The Derivative of a Function*	
4.1 Use of the limit definition of the derivative	4.1.1. Define the derivative of a function as (i) the limit of a difference quotient, (ii) the slope of a tangent line, and (iii) the rate of change (marginal functions). 4.1.2. Use the limit definition of the derivative to determine the derivative of a polynomial of degree 1, 2 or 3, square root and simple rational functions. 4.1.3. Use the limit definition of the derivative to find the slope of the tangent line to a curve at a specific point.

Specific Performance Criteria	**Intermediate Learning Objectives**
4.2 Use of the graph of a function to determine whether a function is differentiable at a point or on an interval	4.2.1. Determine if the derivative of a function exists at a point or on an interval by examining the graph of the function.
4.3 Recognition of the equivalence of various derivative notations	4.3.1. Recognize different notations for the derivative of y with respect to x: y', $f'(x)$, $\frac{dy}{dx}$, $\frac{d}{dx}f(x)$, $D_x y$
4.4 Use of basic differentiation formulas and rules	4.4.1. Recognize when and how to use the basic differentiation formulas: $\frac{d}{dx}[c] = 0$; $\frac{d}{dx}[x^n] = nx^{n-1}$; $\frac{d}{dx}[e^x] = e^x$; $\frac{d}{dx}[\ln x] = \frac{1}{x}$; $\frac{d}{dx}[a^x] = a^x \ln a$; $\frac{d}{dx}[\log_a x] = \frac{1}{x \ln a}$; $\frac{d}{dx}[\sin x] = \cos x$; $\frac{d}{dx}[\cos x] = -\sin x$; $\frac{d}{dx}[\tan x] = \sec^2 x$; $\frac{d}{dx}[\sec x] = \sec x \tan x$; $\frac{d}{dx}[\csc x] = -\csc x \cot x$; $\frac{d}{dx}[\cot x] = -\csc^2 x$ 4.4.2. Recognize when and how to use the following differentiation formulas derived from the chain rule: $\frac{d}{dx}[u^n] = nu^{n-1}u'$; $\frac{d}{dx}[e^u] = e^u u'$; $\frac{d}{dx}[\ln u] = \frac{u'}{u}$; $\frac{d}{dx}[a^u] = a^u(\ln a)u'$; $\frac{d}{dx}[\log_a u] = \frac{u'}{u \ln a}$; $\frac{d}{dx}[\sin u] = \cos u\, u'$; $\frac{d}{dx}[\cos u] = -\sin u\, u'$; $\frac{d}{dx}[\tan u] = \sec^2 u\, u'$; $\frac{d}{dx}[\sec u] = \sec u \tan u\, u'$; $\frac{d}{dx}[\csc u] = -\csc u \cot u\, u'$; $\frac{d}{dx}[\cot u] = -\csc^2 u\, u'$ 4.4.3. Recognize when and how to use the following rules: constant rule, power rule, constant multiple rule, sum and difference rule. 4.4.4. Recognize when and how to use the product, quotient and chain rules.
4.5 Use of differentiation rules to perform implicit and logarithmic differentiation	4.5.1. Recognize when and how to use implicit differentiation. 4.5.2. Recognize when and how to use logarithmic differentiation.
4.6 Evaluation and application of higher order derivatives	4.6.1. Find higher order derivatives.
4.7 Use of derivatives to find the slope of a tangent line to a curve at a point	4.7.1. Use the differentiation rules listed in 4.4.1 and 4.4.2 to find the slope of the tangent line to a curve at a point. 4.7.2. Use the differentiation rules listed in 4.4.1 and 4.4.2 to find the equation of the tangent line to a curve at a point.
5. *Curve Sketching*	
5.1 Use of the derivative and related concepts to analyze the variations of a function and to sketch a graph of the function	5.1.1. Find critical numbers. 5.1.2. Find intervals on which a function is increasing and decreasing using the sign of the first derivative. 5.1.3. Find relative and absolute extrema. 5.1.4. Use the first or second derivative test to decide whether the critical points represent relative maxima or relative minima. 5.1.5. Find inflection points. 5.1.6. Find intervals on which a function is concave up or concave down using the sign of the second derivative. 5.1.7. Use limits to find all vertical and horizontal asymptotes. 5.1.8. Use 5.1.1–5.1.7 to graph polynomial and rational functions.
6. *Optimization and Rate-of-Change Problems*	
6.1 Solution of optimization problems	6.1.1. Represent an optimization word problem in functional form. 6.1.2. Determine the quantity, Q, to be maximized or minimized and identify the variables which are involved. 6.1.3. Draw a diagram, if possible, to illustrate the problem and list any other relationship(s) between the variables. 6.1.4. Express Q as a function of one variable. 6.1.5. Find the derivative of the function for Q obtained in 5.1.4. 6.1.6. Find all the possible critical values by solving the equation $Q' = 0$. 6.1.7. Test the critical value(s) and interval endpoints for absolute maximum or minimum. 6.1.8. Interpret (explain) the results found in the optimization problem.
6.2 Solution of problems involving rates of change	6.2.1. Find the marginal cost, marginal revenue and marginal profit using the derivative. 6.2.2. Find the elasticity of demand function. Evaluate and classify it at a given production level.