Theorems

(1) Every set of vectors containing $\vec{O}$ is LD. i.e. Prove $\left\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_n}, \vec{O}\right\}$ is LD.

(2) If $S = \left\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_n}\right\}$ is LI , then every subset of vectors in S is LI.

in particular if $\left\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_n}\right\}$ is LI, prove $\left\{\vec{v_1}, \vec{v_2}, \vec{v_3}\right\}$ is LI.

(3) If $S = \left\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_n}\right\}$ is LD , prove $\left\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_n}, \overrightarrow{v_{n+1}}\right\}$ is LD.

(4) Span $\left\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_n}\right\}$ is a subspace $\mathbb{R}^n$ if $\vec{v_i}$'s are vectors in $\mathbb{R}^n$.

(5) (a) $S = \left\{x \in \mathbb{R}^n \middle| A\vec{x} = \vec{O}\right\}$ is a subspace of $\mathbb{R}^n$. This is the Null A or the solution space of $A\vec{x} = \vec{O}$.

(b) $S = \left\{x \in \mathbb{R}^n \middle| A\vec{x} = \vec{b}, \vec{b} \neq \vec{O}\right\}$ is not a subspace of $\mathbb{R}^n$.

(6) a set of <u>one</u> non-zero vector is LI.

(7) a set of <u>two</u> non-zero vectors is LD if and only if the 2 vectors are multiples.

(8) a set of vectors is LD if and only if at least one of the vectors = a L.C. of remaining vectors.

(9) If $\left\{\vec{u}, \vec{v}, \vec{w}\right\}$ is a basis for a 3-d subspace S of a vector space V, prove any vector in S = a <u>unique</u> (only one!) L.C. of $\left\{\vec{u}, \vec{v}, \vec{w}\right\}$.

(10) If $\left\{\vec{u}, \vec{v}\right\}$ is LI and $\vec{w} \notin$ Span$\left\{\vec{u}, \vec{v}\right\}$, then $\left\{\vec{u}, \vec{v}, \vec{w}\right\}$ is LI.

(start with equation $c_1\vec{u} + c_2\vec{v} + c_3\vec{w} = \vec{O}$)

(11) If dimension of S is 3, and $\left\{\vec{u}, \vec{v}, \vec{w}\right\}$ spans S , prove $\left\{\vec{u}, \vec{v}, \vec{w}\right\}$ is LI. (S = a subspace or Vector Space)

(12) If dimension of S is 3, and $\left\{\vec{u}, \vec{v}, \vec{w}\right\}$ is LI , prove Span$\left\{\vec{u}, \vec{v}, \vec{w}\right\} = S$ (S = a subspace or V.S.)

(13) If $\left\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_n}\right\}$ is LI and A^{-1} exists, prove $\left\{A\vec{v_1}, A\vec{v_2}, \ldots, A\vec{v_n}\right\}$ is LI.

Theoretical Questions:

(14) If $\left\{\vec{u}, \vec{v}, \vec{w}\right\}$ is LI, (a) prove $\left\{\vec{u} - \vec{v}, \vec{v} - \vec{w}, \vec{w} - \vec{u}\right\}$ is LD ; (b) prove $\left\{\vec{u}, \vec{u} + \vec{v}, \vec{u} + \vec{v} + \vec{w}\right\}$ is LI.

(11) and (12) $\Rightarrow$ if the dimension of a space (subspace or V.S.) is known and you select a basis for the space $\rightarrow$ to prove that your set of selected vectors is a basis for the subspace (or V.S.) , you do not need to prove both spanning and LI $-$ one of these is enough! we usually prove LI - it is easier!