Review Problems # 4

(1) Given $\vec{u}=(3,2,-1)$, $\vec{v}=(1,2,-4)$, $\vec{w}=(0,-6,7)$,

find (a) $\vec{v}\times\vec{w}$; (b) a unit vector in the direction of $\vec{v}\times\vec{w}$; (c) a vector of length 4 in the direction of $\vec{v}\times\vec{w}$; (d) $\vec{u}\times(\vec{v}-2\vec{w})$; (e) the area of the parallelogram determined by $\vec{v}$ and $\vec{w}$;

(f) $\text{Proj}_{\vec{v}}\ \vec{u}$; (g) the component of $\vec{u}$ orthogonal to $\vec{v}$

(2) Given 3 points : P $(2,-6,1)$, Q$(1,1,1)$, R$(4,6,2)$,

find (a) $\angle$ P , $\angle$ Q , $\angle$ R ; (b) the area of $\triangle$ PQR

(3) Find $\vec{u}\bullet(\vec{v}\times\vec{w})$ given $\vec{u}=(1,3,-5)$, $\vec{v}=(3,4,5)$ and $\vec{w}=(1,3,-4)$

(4) Assume that $\vec{u}\bullet(\vec{v}\times\vec{w})=-4$ (Remember that $\vec{u}\bullet(\vec{v}\times\vec{w})$ is a determinant)

Find : (a) $\vec{u}\bullet(\vec{w}\times\vec{v})$; (b) $(\vec{v}\times\vec{w})\bullet\vec{u}$; (c) $\vec{w}\bullet(\vec{u}\times\vec{v})$

(d) $\vec{v}\bullet(\vec{u}\times\vec{w})$; (e) $(\vec{u}\times\vec{w})\bullet\vec{v}$; (f) $\vec{v}\bullet(\vec{w}\times\vec{w})$

(5) Find the volume of the parallelepiped determined by

$\vec{u}=(1,2,1)$, $\vec{v}=(3,1,-2)$ and $\vec{w}=(2,1,3)$

(6) Prove the identities:

(a) $(k\vec{u}+\vec{v})\times\vec{v}=\vec{u}\times(k\vec{v})$; (b) $\vec{u}\bullet(\vec{v}\times\vec{w})=-(\vec{u}\times\vec{w})\bullet\vec{v}$

(c) $\|2\vec{u}-3\vec{v}\|^2=4\|\vec{u}\|^2-12(\vec{u}\bullet\vec{v})+9\|\vec{v}\|^2$

(7) Given $\vec{u}=(2,-1,1)$, $\vec{v}=(1,1,2)$

(a) find the angle between $\vec{u}$ and $\vec{v}$ using the dot product

(b) verify that $\|\vec{u}\times\vec{v}\|=\|\vec{u}\|\|\vec{v}\|\sin\theta$ using $\vec{u}$ and $\vec{v}$ and the angle you found in (a)

(8) (a) Verify that $\|\vec{u}+\vec{v}\|^2+\|\vec{u}-\vec{v}\|^2=2\left(\|\vec{u}\|^2+\|\vec{v}\|^2\right)$

(b) Interpret your result in (a) geometrically. Remember that $\vec{u}+\vec{v}$ and $\vec{u}-\vec{v}$ are the diagonals of the parallelogram determined by $\vec{u}$ and $\vec{v}$.

(9) Find the following intersections or show that there is no intersection

(a) the plane $\begin{cases}4x+12y+3z=24\\2x+5y+10z=20\\2x+\ y+\ 2z=\ 8\end{cases}$ (b) the plane $\begin{cases}3x+6y+4z=18\\3x+3y+4z=15\\3x+2y+4z=14\end{cases}$

(c) the line L : $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5\\-3\\-1\end{pmatrix}+t\begin{pmatrix}-1\\2\\-5\end{pmatrix}$ and the plane $3x+4y-z=-26$; (d) the line L_1 : $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\4\\1\end{pmatrix}+t\begin{pmatrix}4\\1\\0\end{pmatrix}$ and the line L_2 : $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-1\\7\\5\end{pmatrix}+s\begin{pmatrix}12\\6\\3\end{pmatrix}$

(e) the line L : $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5\\-3\\-1\end{pmatrix}+t\begin{pmatrix}-1\\2\\-5\end{pmatrix}$ and the plane $3x-y-z=20$

Review Problems # 4

(10) Find the equation of a plane

(a) containing the position vectors $\vec{u}=\left(1,2,-5\right)$ and $\vec{v}=\left(-2,3,8\right)$

(b) containing the lines in (9 d)

(c) containing the point (5, 3, -2) and the line $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}0\\3\\-7\end{pmatrix}$

(d) containing the point (2, 1, 3) and perpendicular to 2 planes P_1: $4x-2y+2z=5$ and P_2: $3x+3y-6z=12$

(e) containing the point (2, -1, 3) and parallel to the plane P: $4x-y+2z=12$

(f) containing the point (2, 1, -3) and perpendicular to the line $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\5\\-9\end{pmatrix}+t\begin{pmatrix}0\\2\\-3\end{pmatrix}$

(g) containing the point (2, 1, -4) and perpendicular to the line of intersection of the planes $4x+2y+2z=-1$ and $3x+6y+3z=7$

(11) Find the equation of a line:

(a) containing 2 points P_1 $\left(2,-1,-3\right)$ and P_2 $\left(1,2,4\right)$

(b) through (1, -4, 5) and parallel to planes $2x+y-4z=0$ and $-x+2y+3z+1=0$

(c) containing point (1, 2, 3) and perpendicular to plane $3x+y-4z=12$

(d) which is the intersection of the planes P_1: $-3x+2y+z=-5$ and P_2: $7x+3y-2z=-2$

(12) Sketch each of the following: state the normal and show intercepts and traces.

(a) $x=3$; (b) $y=-2$; (c) $3y+2z=6$; (d) $3x+2y=6$

(e) $3x+2z=6$; (f) $3x+2y+4z=24$

(13) Find the perpendicular distance between

(a) the point (1, -1, 3) and the line $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\0\\-1\end{pmatrix}+t\begin{pmatrix}3\\1\\4\end{pmatrix}$

(b) the parallel lines L_1: $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-3\\2\\4\end{pmatrix}+t\begin{pmatrix}3\\1\\4\end{pmatrix}$ and L_2: $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\0\\-1\end{pmatrix}+t\begin{pmatrix}3\\1\\4\end{pmatrix}$

(c) point (1, -1, 3) and the plane $3x-y+4z=20$

(d) the parallel planes P_1: $-3x-y+4z=20$ and P_2: $3x-y+4z=-3$

(e) the skew lines L_1: $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\0\\1\end{pmatrix}+t\begin{pmatrix}2\\1\\-3\end{pmatrix}$ and L_2: $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\1\\-1\end{pmatrix}+t\begin{pmatrix}1\\0\\1\end{pmatrix}$

Review Problems # 4

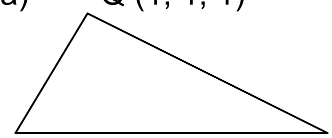
Solutions:

(1) (a) $\vec{v}\times\vec{w} = \left(-10,-7,-6\right)$; (b) $\dfrac{1}{\left\|\vec{v}\times\vec{w}\right\|}\left(\vec{v}\times\vec{w}\right) = \dfrac{1}{\sqrt{185}}\left(-10,-7,-6\right)$; (c) $\dfrac{4}{\sqrt{185}}\left(-10,-7,-6\right)$

(d) $\vec{v}-2\vec{w} = \left(1,14,-18\right)$, $\vec{u}\times\left(\vec{v}-2\vec{w}\right) = \left(-22,53,40\right)$; (e) $\left\|\vec{v}\times\vec{w}\right\| = \sqrt{185}$

(f) $\dfrac{\vec{u}\bullet\vec{v}}{\left\|\vec{v}\right\|^2}\vec{v} = \dfrac{11}{21}\left(1,2,-4\right)$; (g) $\vec{u}-\text{Proj}_{\vec{v}}\ \vec{u} = \dfrac{1}{21}\left(52,20,23\right)$

(2 a) Q (1, 1, 1)



P (2, -6, 1) R (4, 6, 2)

$$\cos P = \frac{\overrightarrow{PQ}\bullet\overrightarrow{PR}}{\left\|\overrightarrow{PQ}\right\|\left\|\overrightarrow{PR}\right\|} = \frac{82}{\sqrt{50}\ \sqrt{149}} \Rightarrow \angle P \approx 18.19^\circ$$

$$\cos Q = \frac{\overrightarrow{QP}\bullet\overrightarrow{QR}}{\left\|\overrightarrow{QP}\right\|\left\|\overrightarrow{QR}\right\|} = \frac{-32}{\sqrt{50}\ \sqrt{35}} \Rightarrow \angle Q \approx 139.9^\circ$$

$$\cos R = \frac{\overrightarrow{RP}\bullet\overrightarrow{RQ}}{\left\|\overrightarrow{RP}\right\|\left\|\overrightarrow{RQ}\right\|} = \frac{67}{\sqrt{149}\ \sqrt{35}} \Rightarrow \angle R \approx 21.91^\circ$$

(2 b) $\overrightarrow{PQ}\times\overrightarrow{PR} = \left(7,1,-26\right) \ \rightarrow$ area $\triangle$ PQR $= \dfrac{1}{2}\left\|\overrightarrow{PQ}\times\overrightarrow{PR}\right\| = \dfrac{1}{2}\sqrt{726} \approx 13.47$ sq. units

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(3) $$\vec{u}\bullet\left(\vec{v}\times\vec{w}\right) = \left|\begin{array}{ccc|c} \boxed{1} & 3 & -5 & \overset{s}{-1} \\ 3 & 4 & 5 & 12 \\ 1 & 3 & -4 & 0 \end{array}\right| = \left|\begin{array}{ccc|c} 1 & 3 & -5 & \overset{s}{-1} \\ 0 & -5 & 20 & 15 \\ 0 & 0 & 1 & 1 \end{array}\right| = -5$$

(4 a) $$\vec{u}\bullet\left(\vec{w}\times\vec{v}\right) = \begin{vmatrix} u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \underset{R_2\leftrightarrow R_3}{=} -\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = -\left(\vec{u}\bullet\left(\vec{v}\times\vec{w}\right)\right) = -(-4) = 4$$

(4 b) $\left(\vec{v}\times\vec{w}\right)\bullet\vec{u} = \vec{u}\bullet\left(\vec{v}\times\vec{w}\right) = -4$ (dot product is commutative)

(4 c) $$\vec{w}\bullet\left(\vec{u}\times\vec{v}\right) = \begin{vmatrix} w_1 & w_2 & w_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \underset{R_1\leftrightarrow R_2}{=} -\begin{vmatrix} u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \underset{R_2\leftrightarrow R_3}{=} (-1)^2\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = \vec{u}\bullet\left(\vec{v}\times\vec{w}\right) = -4$$

(4 d) $$\vec{v}\bullet\left(\vec{u}\times\vec{w}\right) = \begin{vmatrix} v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \underset{R_1\leftrightarrow R_2}{=} -\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = -\left(\vec{u}\bullet\left(\vec{v}\times\vec{w}\right)\right) = -(-4) = 4$$

Review Problems # 4

Solutions:

(4) (e) $\left(\vec{u}\times\vec{w}\right)\bullet\vec{v}=\vec{v}\bullet\left(\vec{u}\times\vec{w}\right)=\begin{vmatrix} v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{vmatrix}\underset{R_1\leftrightarrow R_2}{=}(-1)\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}=(-1)\left(\vec{u}\bullet\left(\vec{v}\times\vec{w}\right)\right)=(-1)(-4)=4$

(f) $\vec{v}\bullet\left(\vec{w}\times\vec{w}\right)=\vec{v}\bullet\overrightarrow{O_v}=0$ (scalar)

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(5) Vol = Absolute value of $\begin{vmatrix} \boxed{1} & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 1 & 3 \end{vmatrix}=$ absolute of $\begin{vmatrix} 1 & 2 & 1 \\ 0 & -5 & -5 \\ 0 & -3 & 1 \end{vmatrix}=$ absolute of $-20=20$ cu. units

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(6 a) $\left(\mathsf{k}\vec{u}+\vec{v}\right)\times\vec{v}=\left(\mathsf{k}\vec{u}\times\vec{v}\right)+\left(\vec{v}\times\vec{v}\right)=\mathsf{k}\left(\vec{u}\times\vec{v}\right)+\overrightarrow{O_v}=\mathsf{k}\left(\vec{u}\times\vec{v}\right)=\vec{u}\times\left(\mathsf{k}\vec{v}\right)$

(6 b) $\vec{u}\bullet\left(\vec{v}\times\vec{w}\right)=\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}\underset{R_1\leftrightarrow R_2}{=}(-1)\begin{vmatrix} v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{vmatrix}=-\left(\vec{v}\bullet\left(\vec{u}\times\vec{w}\right)\right)=-\left(\left(\vec{u}\times\vec{w}\right)\bullet\vec{v}\right)$

dot product is commutative

(6 c) $\left\|2\vec{u}-3\vec{v}\right\|^2=\left(2\vec{u}-3\vec{v}\right)\bullet\left(2\vec{u}-3\vec{v}\right)=\left(2\vec{u}-3\vec{v}\right)\bullet\left(2\vec{u}\right)-\left(2\vec{u}-3\vec{v}\right)\bullet\left(3\vec{v}\right)$

$=4\vec{u}\bullet\vec{u}-6\vec{v}\bullet\vec{u}-6\vec{u}\bullet\vec{v}+9\vec{v}\bullet\vec{v}=4\left\|\vec{u}\right\|^2-12\left(\vec{u}\bullet\vec{v}\right)+9\left\|\vec{v}\right\|^2$

(7 a) cos θ $=\dfrac{\vec{u}\bullet\vec{v}}{\left\|\vec{u}\right\|\left\|\vec{v}\right\|}=\dfrac{3}{\sqrt{6}\ \sqrt{6}}=\dfrac{1}{2}\Rightarrow\ \theta=60^\circ$

(7 b) $\vec{u}\times\vec{v}=\left(-3,-3,3\right)\Rightarrow\left\|\vec{u}\times\vec{v}\right\|=3\left(-1,-1,1\right)=3\sqrt{3}\Rightarrow\left\|\vec{u}\right\|\left\|\vec{v}\right\|$ sin $\theta=\sqrt{6}\sqrt{6}$ sin $60^\circ=6\dfrac{\sqrt{3}}{2}=3\sqrt{3}$

therefore , $\left\|\vec{u}\times\vec{v}\right\|=\left\|\vec{u}\right\|\left\|\vec{v}\right\|$ sin θ

(8 a) $\left\|\vec{u}+\vec{v}\right\|^2+\left\|\vec{u}-\vec{v}\right\|^2=\left(\vec{u}+\vec{v}\right)\bullet\left(\vec{u}+\vec{v}\right)+\left(\vec{u}-\vec{v}\right)\bullet\left(\vec{u}-\vec{v}\right)$

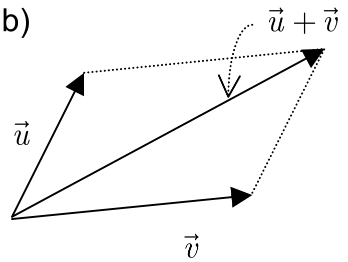
$=\vec{u}\bullet\vec{u}+\vec{v}\bullet\vec{u}+\vec{u}\bullet\vec{v}+\vec{v}\bullet\vec{v}+\vec{u}\bullet\vec{u}-\vec{v}\bullet\vec{u}-\vec{u}\bullet\vec{v}+\vec{v}\bullet\vec{v}$

$=\left\|\vec{u}\right\|^2+2\vec{u}\bullet\vec{v}+\left\|\vec{v}\right\|^2+\left\|\vec{u}\right\|^2-2\vec{u}\bullet\vec{v}+\left\|\vec{v}\right\|^2=2\left(\left\|\vec{u}\right\|^2+\left\|\vec{v}\right\|^2\right)$

Review Problems # 4

Solutions:

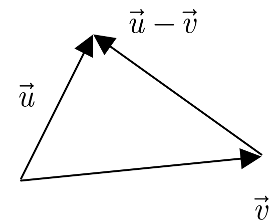
(8 b)



$\vec{u}+\vec{v}$ and $\vec{u}-\vec{v}$ form the diagonals of the parallelogram

sum of squares of diagonals =

2x(sum of squares of sides of parallelogram)



(9 a) $\left|\begin{array}{ccc|c|c} 4 & 12 & 3 & 24 & 43 \\ 2 & 5 & 10 & 20 & 37 \\ 2 & 1 & 2 & 8 & 13 \end{array}\right| \Rightarrow (x,y,z) = \left(\frac{5}{2},\frac{19}{21},\frac{22}{21}\right)$, a single point of intersection

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(9 b) $\left|\begin{array}{ccc|c|c} 3 & 6 & 4 & 18 & 31 \\ 3 & 3 & 10 & 15 & 25 \\ 2 & 1 & 2 & 14 & 23 \end{array}\right| \Rightarrow (x,y,z) = \left(\frac{-4t+12}{3},1,t\right)$, a line of intersection

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(9 c) $3(5-t)+4(-3+2t)-(-1-5t)=-26 \Rightarrow 15-3t-12+8t+1+5t=-26$

$10t=-26-4=-30 \Rightarrow t=-3$; point of intersection : $(x,y,z)=(8,-9,14)$

(9 d) $\begin{cases} 3+4t=-1+12s \\ 4+t=7+6s \\ 1=5+3s \end{cases} \Rightarrow \begin{cases} 3+4t=-1+12s \\ 4+t=7+6s \\ 1=5+3s \end{cases} \Rightarrow \left[\begin{array}{cc|c} \overset{t}{\boxed{1}} & \overset{s}{-6} & 3 \\ 1 & -3 & -1 \\ 0 & -3 & 4 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -6 & 3 \\ 0 & \boxed{3} & -4 \\ 0 & -3 & 4 \end{array}\right] \sim \left[\begin{array}{cc|c} 3 & 0 & -15 \\ 0 & 3 & -4 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{c} t=-5 \\ s=-\frac{4}{3} \end{array}$

$\left.\begin{array}{l} t=-5 \Rightarrow (x,y,z)=(3,4,1)+(-5)(4,1,0)=(-17,-1,1) \\ s=-\frac{4}{3} \Rightarrow (x,y,z)=(-1,7,5)-\frac{4}{3}(12,6,3)=(-17,-1,1) \end{array}\right\}$ point of intersection

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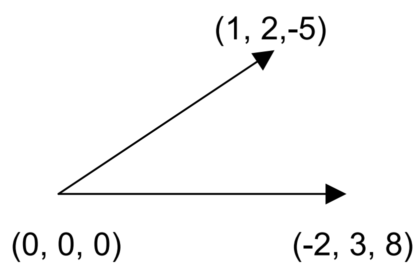
(9 e) $3(5-t)-(-3+2t)-(-1-5t)=20 \Rightarrow 15-3t+3-2t+1+5t=20 \Rightarrow 19=20$ impossible

therefore , L does not intersect P . (L is parallel to P)

(10 a) $\vec{n}=\vec{u}\times\vec{v}=(31,2,7)$

$(31,2,7)\bullet(x-0,y-0,z-0)=0$

$31x+2y+7z=0$



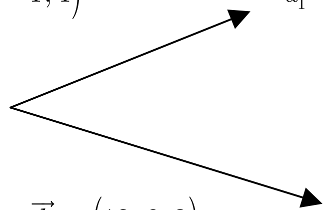
or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} + s\begin{pmatrix} -2 \\ 3 \\ 8 \end{pmatrix}$

<u>**Review Problems # 4**</u>

Solutions:

(10 b) $\vec{n} = k\left(\vec{d_1} \times \vec{d_2}\right) = \left(3,-12,12\right)$; Let $\vec{n} = \left(1,-4,4\right)$

$\vec{d_1} = \left(4,1,0\right)$



$\left(1,-4,4\right) \bullet \left(x-3, y-4, z-1\right) = 0$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + s\begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$

$x - 4y + 4z = -9$

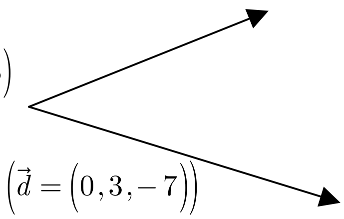
$\vec{d_2} = \left(12,6,3\right)$

(10 c) $\vec{n} = k\left(\overrightarrow{P_0P_1} \times \vec{d}\right) = k\left(8,28,12\right)$; $\overrightarrow{P_0P_1} = \left(8,28,12\right)$; Let $\vec{n} = \left(2,7,3\right)$

$P_1 = \left(5,3,-2\right)$

$\left(2,7,3\right) \bullet \left(x-1, y-2, z-3\right) = 0$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix} + s\begin{pmatrix} 0 \\ 3 \\ -7 \end{pmatrix} + t\begin{pmatrix} 4 \\ 1 \\ -5 \end{pmatrix}$

$P_0\left(1,2,3\right)$



$2x + 7y + 3z = 25$

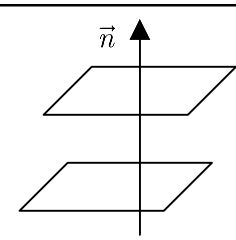
L : $\left(\vec{d} = \left(0,3,-7\right)\right)$

(10 d) $\vec{n} \perp \vec{n_1}$ and $\vec{n} \perp \vec{n_2}$ (2 planes are $\perp$ if and only if their normals are $\perp$) ;

$\vec{n} = k\left(\vec{n_1} \times \vec{n_2}\right) = \left(6,30,18\right) = 6\left(1,5,3\right)$; Let : $\vec{n} = \left(1,5,3\right)$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

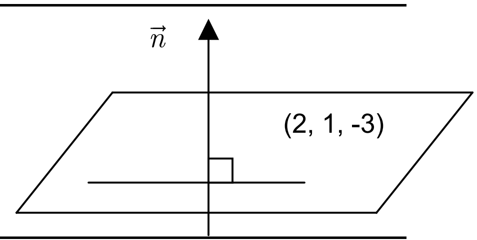
$\left(1,5,3\right) \bullet \left(x-2, y-1, z-3\right) = 0 \Rightarrow x + 5y + 3z = 16$

(10 e) $\vec{n} = \vec{n_1} = \vec{n_2} = \left(4,-1,2\right)$; (parallel planes have equal normals)



$\left(4,-1,2\right) \bullet \left(x-2, y+1, z-3\right) = 0 \Rightarrow 4x - y + 2z = 15$

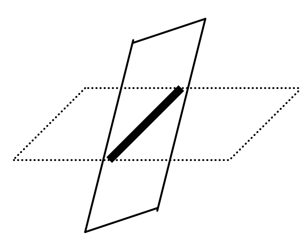
(10 f) $\vec{d} = \vec{n} = \left(0,2,-3\right)$; (since L $\perp$ P and $\vec{n} \perp$ P)



$\left(0,2,-3\right) \bullet \left(x-2, y-1, z+3\right) = 0 \Rightarrow 2y - 3z = 11$

(10 g) the line (lies on both planes) of intersection is to $\perp$ both normals $\vec{n_1}$ and $\vec{n_2}$

so that $\vec{d} = k\left(\vec{n_1} \times \vec{n_2}\right) = \left(-6,-6,18\right) = -6\left(1,1,-3\right)$; Let : $\vec{d} = \left(1,1,-3\right)$ or



$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \Rightarrow x + y - 3z = 15$; alternative solution:

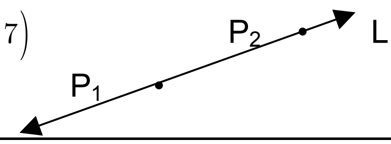
Find the line of intersection for the unknown plane $\left[\begin{array}{ccc|c} 4 & 2 & 2 & -1 \\ 3 & 6 & 3 & 7 \end{array}\right] \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = (\ ) + t\left(\vec{d}\right)$ and let $\vec{n} = \vec{d}$

Review Problems # 4

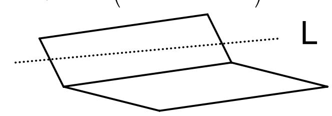
Solutions:

(11 a) $\vec{d} = \text{k}(P_1 - P_2) = \text{k}(1,-3,-7)$; Let $\vec{d} = (1,-3,-7)$ (or any multiple of this vector)

L: $(x,y,z) = (2,-1,-3) + t(1,-3,-7)$ or $= (1,2,4) + t(1,-3,-7)$

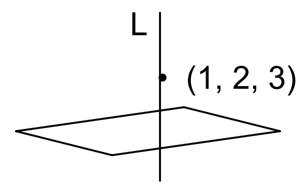


(11 b) L $\perp$ 2 planes $\rightarrow$ L $\perp$ to both normals $\rightarrow$ $\vec{d} = \text{k}(\vec{n_1} \times \vec{n_2}) = \text{k}(-11,2,-5)$; Let $\vec{d} = (11,-2,5)$



L: $(x,y,z) = (1,-4,5) + t(11,-2,5)$

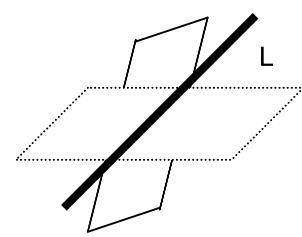
(11 c) $\vec{n} = (3,1,-4) = \vec{d}$



$(x,y,z) = (1,2,3) + t(3,1,-4)$

(11 d) $\begin{bmatrix} \boxed{3} & -2 & -1 & 15 & \overset{s}{5} \\ 7 & 3 & -2 & -2 & 6 \end{bmatrix} \sim \begin{bmatrix} \overset{x}{23} & \overset{y}{0} & \overset{z=t}{-7} & 11 & \overset{s}{27} \\ 0 & 23 & 1 & -41 & -17 \end{bmatrix} \Rightarrow (x,y,z) = \left(\frac{7t+11}{23}, \frac{-t-41}{23}, t\right)$

$(x,y,z) = \left(\frac{7t+11}{23}, \frac{-t-41}{23}, t\right) = \left(\frac{11}{23}, -\frac{41}{23}, 0\right) + t\left(\frac{7}{23}, -\frac{1}{23}, 1\right)$ is one possible solution!



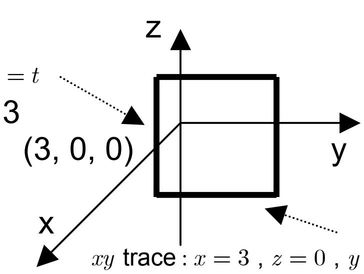
(12 a) x = 3 plane parallel to front wall (yz plane)

$\vec{n} = (1,0,0)$

xy trace : $x = 3$, $y = 0$, $z = t$

traces: y = 0 → x = 3

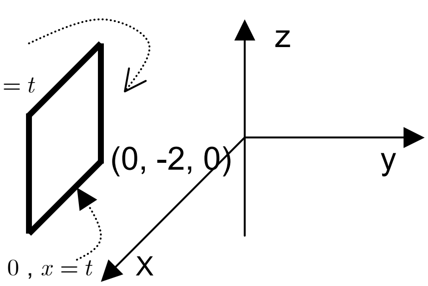
z = 0 → x = 3



xy trace : $x = 3$, $z = 0$, $y = t$

(12 b) $\vec{n} = (0,1,0)$ plane parallel to side wall (xz plane)

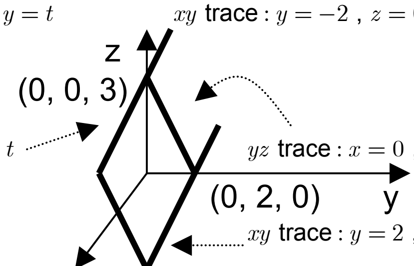
yz trace : $x = 0$, $y = -2$, $z = t$



xy trace : $y = -2$, $z = 0$, $x = t$

(12 c) $\vec{n} = (0,3,2)$ plane parallel to x-axis

xz trace : $z = 3$, $y = 0$, $x = t$



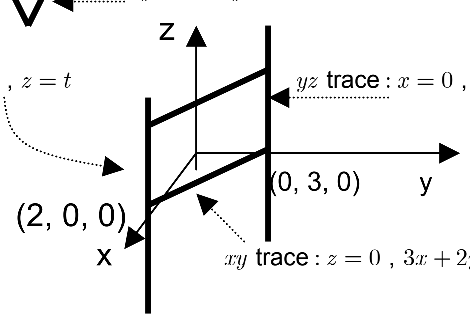
yz trace : $x = 0$, $3y + 2z = 6 \rightarrow \left(0, 2 - \frac{2}{3}t, t\right)$

xy trace : $y = 2$, $z = 0$, $x = t$

(12 d) $\vec{n} = (3,2,0)$ plane parallel to z-axis

xz trace : $x = 2$, $y = 0$, $z = t$

yz trace : $x = 0$, $y = 3$, $z = t$



xy trace : $z = 0$, $3x + 2y = 6 \rightarrow \left(2 - \frac{2}{3}t, t, 0\right)$

Review Problems # 4

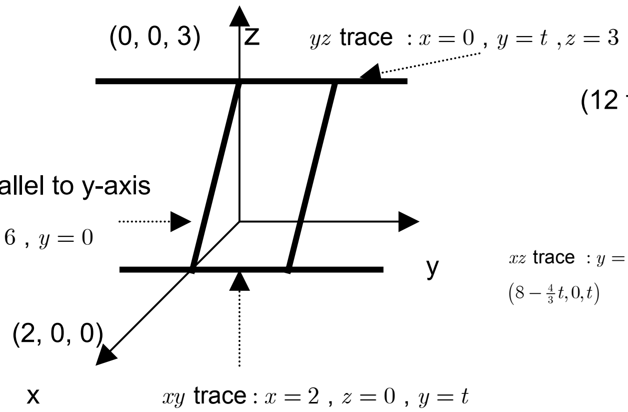
Solutions:

(12 e) x = 3 plane

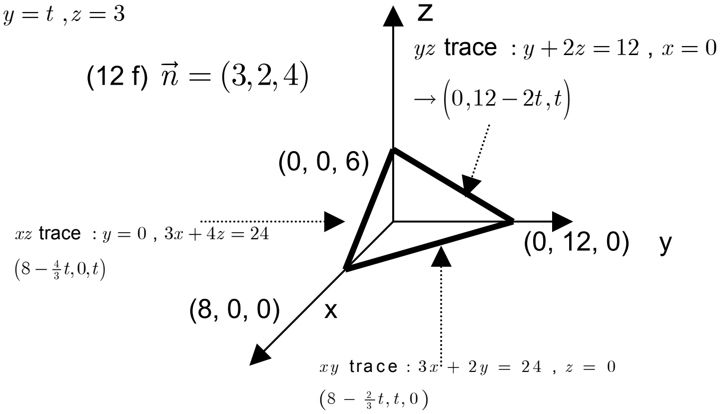
$\vec{n} = (3,0,2)$ parallel to y-axis

xz trace $: 3x + 2z = 6 \ , \ y = 0$

$\rightarrow \left(2 - \frac{2}{3}t, 0, t\right)$

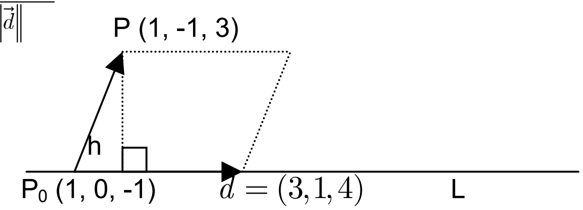


(12 f) $\vec{n} = (3,2,4)$



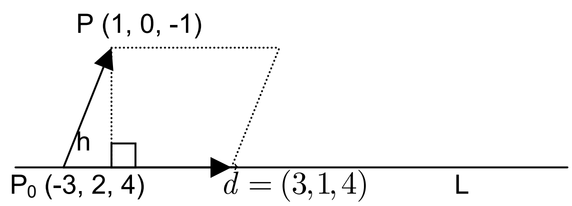
(13 a) h = $\dfrac{\text{area of parallelogram determined by } \overrightarrow{P_0P} \text{ and } \vec{d}}{\|\vec{d}\|} = \dfrac{\|\overrightarrow{P_0P} \times \vec{d}\|}{\|\vec{d}\|}$

h $= \dfrac{\|(-8, 12, 3)\|}{\sqrt{26}} = \dfrac{\sqrt{217}}{\sqrt{26}} \approx 2.89$ units

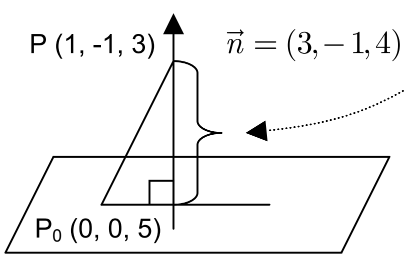


(13 b) h $= \dfrac{\|\overrightarrow{P_0P} \times \vec{d}\|}{\|\vec{d}\|}$

h $= \dfrac{\|(-3, -31, 10)\|}{\sqrt{26}} = \dfrac{\sqrt{1070}}{\sqrt{26}} \approx 6.42$ units



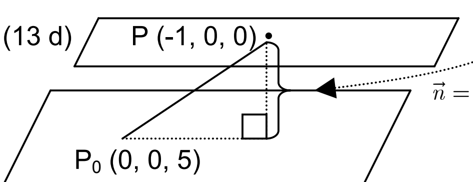
(13 c)



distance = $\|\text{Proj}_{\vec{n}} \overrightarrow{P_oP}\| = \left\| \dfrac{\overrightarrow{P_0P} \bullet \vec{n}}{\|\vec{n}\|^2} \vec{n} \right\|$

$= \left\| \dfrac{(1, -1, -2) \bullet (3, -1, 4)}{(\sqrt{26})^2} (3, -1, 4) \right\| = \dfrac{4}{26}\sqrt{26} = \dfrac{4}{\sqrt{26}} \approx 0.78$ unit

(13 d)



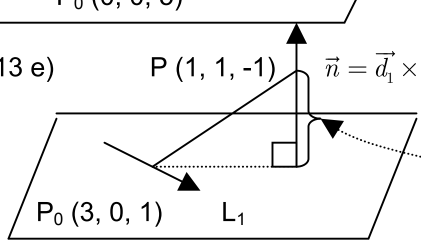
$\vec{n} = (3, -1, 4)$

distance = $\|\text{Proj}_{\vec{n}} \overrightarrow{P_oP}\| = \left\| \dfrac{\overrightarrow{P_0P} \bullet \vec{n}}{\|\vec{n}\|^2} \vec{n} \right\|$

distance $= \left\| \dfrac{(-1, 0, -5) \bullet (3, -1, 4)}{(\sqrt{26})^2} (3, -1, 4) \right\| = \dfrac{23}{26}\sqrt{26} = \dfrac{23}{\sqrt{26}} \approx 4.51$ units

(13 e)

$\vec{n} = \vec{d_1} \times \vec{d_2} = (2, 1, -3) \times (1, 0, 1) = (1, -5, -1)$



distance = $\|\text{Proj}_{\vec{n}} \overrightarrow{P_oP}\| = \left\| \dfrac{(-2, 1, -2) \bullet (1, -5, -1)}{(\sqrt{27})^2} (1, -5, -1) \right\| = \dfrac{5}{\sqrt{27}} \approx 0.96$ unit