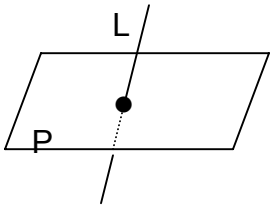
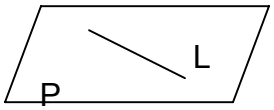
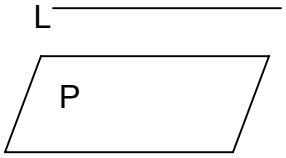
Intersection of a LINE and a PLANE



single point
(called a piercing point)



L lies on P ($\vec{d} \perp \vec{n}$)
(infinite intersection)



L parallel to P and L ∩ P = ∅
($\vec{d} \perp \vec{n}$) (no intersection)

(1) Find the intersection of the line L with each of the planes P_1 , P_2 , P_3

$$L: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \quad ; \quad \begin{array}{l} P_1 : x - 2y + z = 5 \\ P_2 : x - 2y + z = 0 \\ P_3 : x + 2y - z = 5 \end{array}$$

(2) Show that $L: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 8 \\ -2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ is parallel to the plane $P : 5x - y + 3z = -4$

(3) Show that $L : (x, y, z) = (2, 8, -2) + t\,(1, 2, -1)$ lies on the plane $P : 5x - y + 3z = -4$

(4) Find the point of intersection (if any) of the plane $P : 5x - y + 3z = -4$ and

the line $L: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 1 \\ -3 \\ 7 \end{pmatrix}$

Answers:

(1) L parallel to P_1 , L lies on P_2 , L intersects P_3 at $\left(\frac{5}{2}, -1, -\frac{9}{2}\right)$

(3) 5 (2+t) – (8+2t) +3 (-2-t) = -4 gives -4 = -4 for all t

(4) $t = -\frac{25}{29} \Rightarrow (x, y, z) = \left(\frac{4}{29}, \frac{46}{29}, -\frac{30}{29}\right)$