Elementary Matrices

(1) Are the following matrices elementary or not ? Justify your answer.

$$A=\begin{pmatrix}1 & -2 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix};\ B=\begin{pmatrix}0 & 1 & 2\\ 1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix};\ C=\begin{pmatrix}4 & 0 & 0\\ 0 & -5 & 0\\ 0 & 0 & 1\end{pmatrix};\ D=\begin{pmatrix}4 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix};\ E=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 3\\ 0 & 0 & 1\end{pmatrix};\ F=\begin{pmatrix}1 & 0\\ -5 & 1\end{pmatrix}$$

(2) Find the inverse of each of the following elementary matrices

$$A=\begin{pmatrix}1 & 0\\ -4 & 1\end{pmatrix};\ B=\begin{pmatrix}5 & 0\\ 0 & 1\end{pmatrix};\ C=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix};\ D=\begin{pmatrix}1 & 3 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix};\ E=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{pmatrix};\ F=\begin{pmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{pmatrix}$$

(3) Given $A=\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{pmatrix};\ B=\begin{pmatrix}4 & 5 & 6\\ 1 & 2 & 3\end{pmatrix};\ C=\begin{pmatrix}1 & 2 & 3\\ 0 & -3 & -6\end{pmatrix};\ D=\begin{pmatrix}1 & 2 & 3\\ 0 & 1 & 2\end{pmatrix}$,

find elementary matrices E, F, G and their inverses such that

(a) $EA=B$; (b) $FA=C$; (c) $GC=D$; (d) $A=E^{-1}B$; (e) $A=F^{-1}C$; (f) $C=G^{-1}D$

(4) Given $A=\begin{pmatrix}4 & -3\\ 2 & 4\end{pmatrix}$, (a) write A^{-1} as a product of elementary matrices

(b) write A as a product of elementary matrices.

(5) Repeat #4 for $A=\begin{pmatrix}0 & 2 & 6\\ 1 & 0 & -2\\ 0 & 0 & -5\end{pmatrix}$

(6) Given $A=\begin{pmatrix}4 & 5 & 6\\ 1 & 2 & 3\end{pmatrix}$, using EROs, reduce A to an RREF $=R$.

Find $E_1, E_2, \ldots, E_k$ such that $E_k E_{k-1} \ldots E_1 A = R$

and $E_1^{-1}, E_2^{-1}, \ldots, E_k^{-1}$ such that $A=\left(E_k \ldots E_1\right)^{-1} R$

Answer :

(6) $\begin{pmatrix}1 & -2\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1/3\end{pmatrix}\begin{pmatrix}1 & 0\\ -4 & 1\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}A = R$ is one possibility

$$A=\left(E_4 E_3 E_2 E_1\right)^{-1} R = E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1} R = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\ 4 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -3\end{pmatrix}\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix} R$$