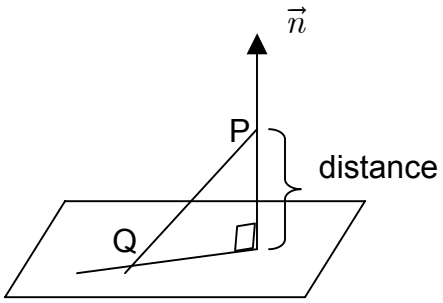
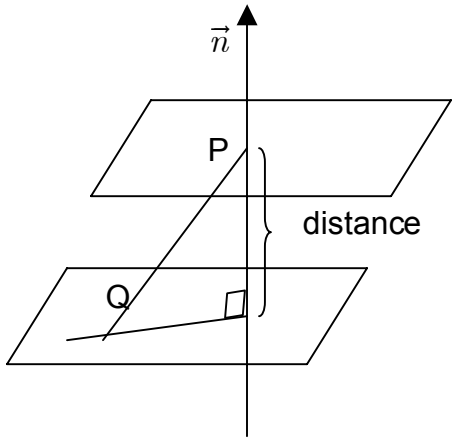
Distance Formulas

(1) Distance between a point and a plane

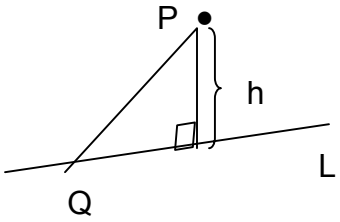


distance = $\| \text{Proj}_{\vec{n}} \, PQ \|$

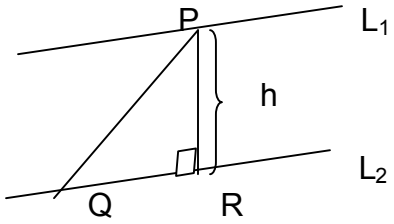
Distance between 2 parallel planes



(2) Distance between a point and a line



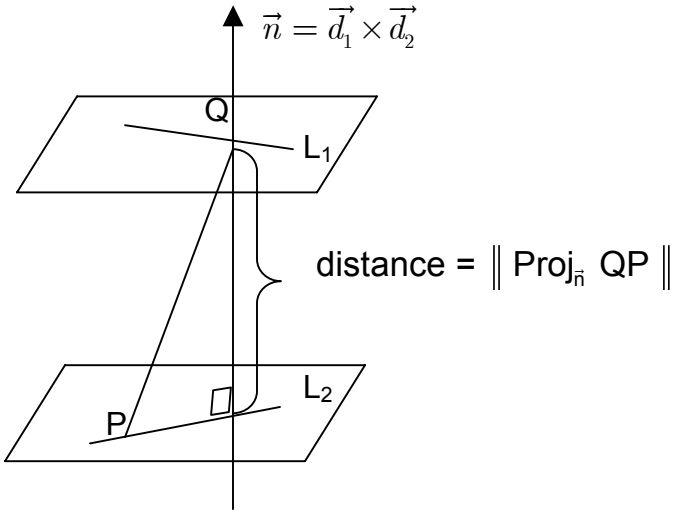
Distance between 2 parallel lines



$$h = \frac{\text{area of parallelogram } \left(\vec{d} \text{ and } \overrightarrow{QP} \right)}{\text{base}} = \frac{\text{twice area of } \triangle PQR}{\text{base}}$$

$$\tfrac{1}{2} \, b \, h = \tfrac{1}{2} \left\| \text{Proj}_{\vec{n}} \, \overrightarrow{QP} \right\| \Rightarrow h = \frac{\left\| \overrightarrow{QP} \times \vec{d} \right\|}{\text{base}} = \frac{\left\| \overrightarrow{QP} \times \vec{d} \right\|}{\left\| \vec{d} \right\|}$$

(3) Distance between Skew Lines (skew lines sit on parallel planes)



DISTANCES

(1) Find the distance from P (1, 2, 3) to a line L where

$$L: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} \quad \left(\text{distance} = \frac{2\sqrt{26}}{\sqrt{17}} \approx 2.47 \text{ units} \right)$$

(2) Find the distance between the parallel lines L_1 and L_2

$$L_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} ; L_2: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} \quad \left(\text{distance} = \frac{\sqrt{179}}{\sqrt{30}} \approx 2.44 \text{ units} \right)$$

(3) Find the distance between the skew lines L_1 and L_2

$$L_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} ; L_2: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + s \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \quad \left(\text{distance} = \frac{4}{\sqrt{171}} \approx 0.31 \text{ unit} \right)$$

Ex 3.5 # 39 , 40

In class : (1) 39 (b) ; (2) 40 (b)

(3) Find the distance between the point P (3, 1, -2) and the line L

$$L: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix}$$

(4) Find the distance between the skew lines

$$L_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 5 \\ -1 \\ 4 \end{pmatrix} ; L_2: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$$