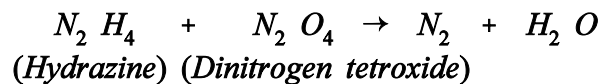


## Balancing Chemical Equations

An example:

Consider the following chemical reaction:



Let:  $x_1$  = number of molecules of  $N_2H_4$  in the reaction  
 $x_2$  = number of molecules of  $N_2O_4$  in the reaction  
 $x_3$  = number of molecules of  $N_2$  in the reaction  
 $x_4$  = number of molecules of  $H_2O$  in the reaction

so that:  $x_1 N_2 H_4 + x_2 N_2 O_4 \rightarrow x_3 N_2 + x_4 H_2 O$

Number of atoms of Nitrogen:  $2x_1 + 2x_2 = 2x_3$  or  $x_1 + x_2 = x_3$

Number of atoms of Hydrogen:  $4x_1 = 2x_4$  or  $2x_1 = x_4$

Number of atoms of Oxygen:  $4x_2 = x_4$

The resulting system of equations is:

$$\left\{ \begin{array}{l} x_1 + x_2 - x_3 = 0 \\ 2x_1 - x_4 = 0 \\ 4x_2 - x_4 = 0 \end{array} \right.$$

$$\begin{array}{l} \left[ \begin{array}{ccccc} 1 & 1 & -1 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \\ 0 & 4 & 0 & -1 & 0 \end{array} \right] \Rightarrow \left[ \begin{array}{ccccc} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & 1 & -3/4 & 0 \end{array} \right] \Rightarrow \\ x_1 = \frac{1}{2}t \\ x_2 = \frac{1}{4}t \\ x_3 = \frac{3}{4}t \\ x_4 = t \end{array}$$

There are many infinitely many choices for  $t$ . Any value of  $t$  such that  $x_1, x_2, x_3, x_4$  are natural numbers (i.e. 1, 2, 3, ...) will suffice.

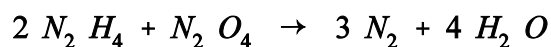
$$x_1 = 2$$

$$x_2 = 1$$

For example, let  $t = 4$ , so that

$$x_3 = 3$$

$$x_4 = 4$$



### Problems

Balance the following chemical equations. Set up the appropriate system of equations and use an augmented matrix to solve the system. Define all variables in words.

