BASIS

(1) Find a basis for each of the following subspaces and state the dimension of each.

(a) $S_1 =$ Span $\left\{(1,1,-3),(2,2,-6),(-4,-4,12)\right\}$ (b) $S_2 = \left\{(x,y,z)\in\mathbb{R}^3 \,\middle|\, y=0\right\}$

(c) $S_3 = \left\{(x,y)\in\mathbb{R}^2 \,\middle|\, x+2y=0\right\}$ (d) $S_4 =$ Span $\left\{(1,1,2),(-1,2,5),(0,3,7),(1,4,9)\right\}$

(e) $S_5 = \left\{(x,y,z)\in\mathbb{R}^3 \,\middle|\, 2x+y-3z=0\right\}$ (f) $S_6 =$ Span $\left\{(1,1,-3),(2,2,-6),(5,6,9)\right\}$

(2) None of the following is a basis for $\mathbb{R}^3$. Why not ?

(a) $\left\{(5,7,-2),(-4,6,9)\right\}$ (b) $\left\{(0,0,0),(1,2,3),(4,5,6)\right\}$

(c) $\left\{(3,5,-7),(-2,4,1),(5,1,-8)\right\}$ (d) $\left\{(1,1,0),(0,1,1),(1,0,1),(1,1,1)\right\}$

(3) (a) What is the standard basis for $\mathbb{R}^4$?
(b) Find another basis for $\mathbb{R}^4$.

(4) Find a basis for the following subspaces. State the dimension of S.

(a) $S=\left\{(x,0)\in\mathbb{R}^2\right\}$ (b) $S=\left\{(x,y)\in\mathbb{R}^2 \,\middle|\, x+2y=0\right\}$ (c) $S=\left\{(x,y,z)\in\mathbb{R}^3 \,\middle|\, z=0\right\}$

(d) $S=\left\{(x,y,z)\in\mathbb{R}^3 \,\middle|\, x+y-z=0\right\}$ (e) $S=\left\{(x,y,z)\in\mathbb{R}^3 \,\middle|\, x-y-z=0\right\}$

(f) $S=\left\{(x,y,z)\in\mathbb{R}^3 \,\middle|\, 4x+y-z=0\right\}$ (g) $S=\left\{(x,x,x)\in\mathbb{R}^3\right\}$

(h) $S=\left\{(x,y,z,w)\in\mathbb{R}^4 \,\middle|\, x+3y=0, z+w=0\right\}$

(i) $S=\left\{(x,y,z,w)\in\mathbb{R}^4 \,\middle|\, x+2y-z+5w=0\right\}$

(j) $S=\left\{(x,y,z,w)\in\mathbb{R}^4 \,\middle|\, x+y=z+w\right\}$ (k) $S=\left\{(a+b,a-b,a,b)\in\mathbb{R}^4\right\}$

(l) $S=\left\{(a,a+b,a-b,b)\in\mathbb{R}^4\right\}$ (m) $S=\left\{(a-b,b+c,a,b+c)\in\mathbb{R}^4\right\}$

(n) $S=\left\{(0,0,0)\in\mathbb{R}^3\right\}$ (o) $\mathbb{R}^3$ (p) $\mathbb{R}^4$

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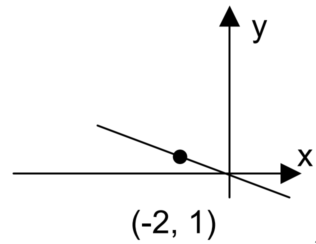
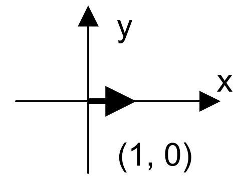
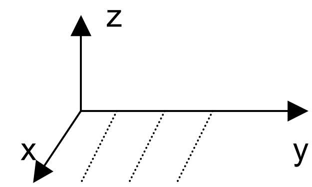
(4 a) basis: $\left\{\left(1,0\right)\right\}$; d = 1 → this is a line in $\mathbb{R}^2$ (the x-axis)



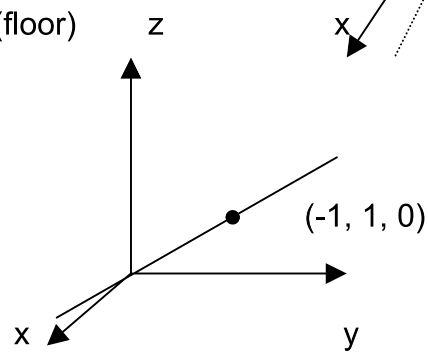


(4 b) basis: $\left\{\left(-2,1\right)\right\}$; d = 1 → a line in $\mathbb{R}^2$

(4 c) basis: $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix}\right\}$; d = 2 → a plane in $\mathbb{R}^3$ → xy plane (floor)



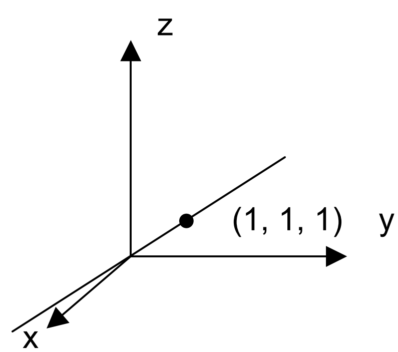
(4 d) basis: $\left\{\begin{pmatrix}-1\\1\\0\end{pmatrix}\right\}$; d = 1 → this is a line in $\mathbb{R}^3$



(4 e) basis: $\left\{\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}1\\0\\1\end{pmatrix}\right\}$; d = 2 → plane in $\mathbb{R}^3$

(4 f) basis: $\left\{\begin{pmatrix}0\\1\\1\end{pmatrix},\begin{pmatrix}1\\0\\4\end{pmatrix}\right\}$; d = 2 → plane in $\mathbb{R}^3$

(4 g) basis: $\left\{\begin{pmatrix}1\\1\\1\end{pmatrix}\right\}$; d = 1 → line in $\mathbb{R}^3$



(4 h) basis: $\left\{\begin{pmatrix}-3\\1\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\-1\\1\end{pmatrix}\right\}$; d = 2 ; $\begin{pmatrix}x\\y\\z\\w\end{pmatrix}=\begin{pmatrix}-3t\\t\\-s\\s\end{pmatrix}$; a 2-dimensional subspace of $\mathbb{R}^4$

(4 i) basis: $\left\{\begin{pmatrix}-2\\1\\0\\0\end{pmatrix},\begin{pmatrix}1\\0\\1\\0\end{pmatrix},\begin{pmatrix}-5\\0\\0\\1\end{pmatrix}\right\}$; d = 3 ; $\begin{pmatrix}x\\y\\z\\w\end{pmatrix}=\begin{pmatrix}-2r+s-5t\\r\\s\\t\end{pmatrix}=r\begin{pmatrix}-2\\1\\0\\0\end{pmatrix}+s\begin{pmatrix}1\\0\\1\\0\end{pmatrix}+t\begin{pmatrix}-5\\0\\0\\1\end{pmatrix}$;

a 3-dimensional subspace of $\mathbb{R}^4$

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(4 j) basis: $\left\{\begin{pmatrix}-1\\1\\0\\0\end{pmatrix},\begin{pmatrix}1\\0\\1\\0\end{pmatrix},\begin{pmatrix}1\\0\\0\\1\end{pmatrix}\right\}$; d = 3 ; $\begin{pmatrix}x\\y\\z\\w\end{pmatrix}=\begin{pmatrix}-r+s+t\\r\\s\\t\end{pmatrix}=r\begin{pmatrix}-1\\1\\0\\0\end{pmatrix}+s\begin{pmatrix}1\\0\\1\\0\end{pmatrix}+t\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$;

a 3-dimensional subspace of $\mathbb{R}^4$

(4 k) basis: $\left\{\begin{pmatrix}1\\1\\1\\0\end{pmatrix},\begin{pmatrix}1\\-1\\0\\1\end{pmatrix}\right\}$; d = 2 ; $\begin{pmatrix}x\\y\\z\\w\end{pmatrix}=\begin{pmatrix}a+b\\a-b\\a\\b\end{pmatrix}=a\begin{pmatrix}1\\1\\1\\0\end{pmatrix}+b\begin{pmatrix}1\\-1\\0\\1\end{pmatrix}$; a 2-dimensional subspace of $\mathbb{R}^4$

(4 l) basis: $\left\{\begin{pmatrix}1\\1\\1\\0\end{pmatrix},\begin{pmatrix}0\\1\\-1\\1\end{pmatrix}\right\}$; d = 2 ; $\begin{pmatrix}x\\y\\z\\w\end{pmatrix}=\begin{pmatrix}a\\a+b\\a-b\\b\end{pmatrix}=a\begin{pmatrix}1\\1\\1\\0\end{pmatrix}+b\begin{pmatrix}0\\1\\-1\\1\end{pmatrix}$; a 2-dimensional subspace of $\mathbb{R}^4$

(4 m) basis: $\left\{\begin{pmatrix}1\\0\\1\\0\end{pmatrix},\begin{pmatrix}-1\\1\\0\\1\end{pmatrix},\begin{pmatrix}0\\1\\0\\1\end{pmatrix}\right\}$; d = 3 ; $\begin{pmatrix}x\\y\\z\\w\end{pmatrix}=\begin{pmatrix}a-b\\b+c\\a\\b+c\end{pmatrix}=a\begin{pmatrix}1\\0\\1\\0\end{pmatrix}+b\begin{pmatrix}-1\\1\\0\\1\end{pmatrix}+c\begin{pmatrix}0\\1\\0\\1\end{pmatrix}$;

a 3-dimensional subspace of $\mathbb{R}^4$

(4 n) d = 0 , no basis

(4 o) standard basis: $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$; d = 3 ; any 3 vectors $\left\{\vec{u},\vec{v},\vec{w}\right\}$ such that $\det\left(\vec{u}\,\vec{v}\,\vec{w}\right)\neq 0$

will form a basis for $\mathbb{R}^3$

(4 p) standard basis: $\left\{\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right\}$; d = 4 ; any 4 vectors $\left\{\vec{u},\vec{v},\vec{w},\vec{x}\right\}$ such that

$\det\left(\vec{u}\,\vec{v}\,\vec{w}\,\vec{x}\right)\neq 0$ will form a basis for $\mathbb{R}^4$

Text : 5.4 ; 1a , b , 2 , 3 , 17 , 18 , 20 , 21 , 22