LU - Decomposition

LU Procedure :

Use only EROs : $kR_i + R_j \rightarrow R_j$; $k \neq 0$, $kR_i \rightarrow R_i$; NO row interchanges

(1) Start with Row 1 : use $kR_i \rightarrow R_i$ to get a leading one in the a_{11} position.

(2) Use your leading one to get zeros below it ($kR_i + R_j \rightarrow R_j$)

(3) Go to Row 2 : use $kR_i \rightarrow R_i$ to get a leading one in the a_{22} position.

(4) Use this leading one as a pivot to get zeros below it . ($kR_i + R_j \rightarrow R_j$)

(5) Repeat until A has been reduced to U .

(6) Obtain L from I using inverse elementary operations.

Given the following system of equations $A\vec{x} = \vec{b}$, find an LU - decomposition for A , check that LU $=$ A , use the LU - decomposition to solve $A\vec{x} = \vec{b}$.

(1) $\begin{pmatrix} 3 & -6 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(2) $\begin{pmatrix} 3 & -6 & -3 \\ 2 & 0 & 6 \\ -4 & 7 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -3 \\ -22 \\ 3 \end{pmatrix}$

(3) $\begin{pmatrix} 2 & 8 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

(4) $\begin{pmatrix} -5 & -10 \\ 6 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -10 \\ 19 \end{pmatrix}$

(5) $\begin{pmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ -1 & 5 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 6 \end{pmatrix}$

(6) $\begin{pmatrix} -3 & 12 & -6 \\ 1 & -2 & 2 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -33 \\ 7 \\ -1 \end{pmatrix}$

(7) $\begin{pmatrix} 5 & 5 & 10 \\ -8 & -7 & -9 \\ 0 & 4 & 26 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$

(8) $\begin{pmatrix} -1 & -3 & -4 \\ 3 & 10 & -10 \\ -2 & -4 & 11 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -6 \\ -3 \\ 9 \end{pmatrix}$

Answers :

(1) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$; (2) $\begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}$; (3) $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$; (4) $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$; (5) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$; (6) $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$; (7) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$; (8) $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$