Math - Calculus II SERIES (Review)

Determine whether the series is absolutely convergent, conditionally convergent or divergent. Justify your conclusions.

(1) $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$; absolutely convergent, compare with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(2) $\sum_{n=1}^{\infty} \frac{2n}{5n-3}$; divergent by (N.T.T.) ; (3) $\sum_{n=1}^{\infty} \frac{1}{5+e^{-n}}$; divergent by (N.T.T.)

(4) $\sum_{n=1}^{\infty} \frac{1}{3^{2n}}$; absolutely convergent, use RootT or RatioT or G.S.

(5) $\sum_{n=1}^{\infty} \left[\left(\frac{2}{3}\right)^n + \left(\frac{3}{2}\right)^n \right]$; divergent the second G.S.

(6) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n^2}\right)$; divergent by (N.T.T.)

(7) $\sum_{n=1}^{\infty} \frac{2}{n+2}$; divergent, compare with $\sum_{n=1}^{\infty} \frac{1}{n}$

(8) $\sum_{n=1}^{\infty} \frac{1}{4(n+1)^2}$; absolutely convergent, compare with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(9) $\sum_{n=1}^{\infty} \left[\frac{1}{2n} + \left(-\frac{1}{2}\right)^n \right]$; first series is divergent, compare with $\frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n}$

(10) $\sum_{n=2}^{\infty} \left[\frac{n}{\ln(n)} + \left(-\frac{2}{3}\right)^n \right]$; first series is divergent by (N.T.T.)

(11) $\sum_{n=1}^{\infty} \frac{1}{n!}$; absolutely convergent by RatioT

(12) $\sum_{n=1}^{\infty} \frac{1}{n\ 2^n}$; absolutely convergent, compare with $\sum_{n=1}^{\infty} \frac{1}{2^n}$ or use RatioT

(13) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$; absolutely convergent by RatioT ; (14) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$; divergent by RatioT

(15) $\sum_{n=1}^{\infty} \frac{n!}{n^2}$; divergent by RatioT

(16) $\sum_{n=1}^{\infty} \frac{2}{2^n - 1}$; absolutely convergent, compare with $\sum_{n=1}^{\infty} \frac{1}{2^n}$ or use RatioT

(17) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$; divergent, compare with p-series $\sum_{n=1}^{\infty} \frac{1}{n^{1/5}}$

(18) $\sum_{n=1}^{\infty} \frac{|\sec(n)|}{n}$; divergent, compare with $\sum_{n=1}^{\infty} \frac{1}{n}$

(19) $\sum_{n=1}^{\infty} \frac{\left(\cos \frac{\pi}{n}\right) + 2}{3^n}$; absolutely convergent, compare with $3 \sum_{n=1}^{\infty} \frac{1}{3^n}$

(20) $\sum_{n=1}^{\infty} \frac{1}{(n^2+3)^{2/3}}$; absolutely convergent, compare with $\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$

(21) $\sum_{n=1}^{\infty} \frac{n^2}{4n^3+1}$; divergent, compare with $\sum_{n=1}^{\infty} \frac{1}{n}$

(22) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2}}$; absolutely convergent, compare with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

(23) $\sum_{n=1}^{\infty} \frac{\sqrt{n+2}}{n^3+1}$; absolutely convergent, compare with $\sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$

(24) $\sum_{n=1}^{\infty} \frac{1}{n + \sqrt{n}}$; divergent, compare with $\sum_{n=1}^{\infty} \frac{1}{n}$

(25) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^2}$; absolutely convergent, compare with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ or use I.T.

(26) $\sum_{n=2}^{\infty} \frac{1}{n\ (\ \ln(n)\ )^{3/2}}$; absolutely convergent by I.T.

(27) $\sum_{n=1}^{\infty} n^2\ e^{-n}$; absolutely convergent by RatioT or I.T.

(28) $\sum_{n=1}^{\infty} \frac{(n+1)!}{(n-1)!\ 2^n}$; absolutely convergent by RatioT

(29) $\sum_{n=1}^{\infty} \arctan(n)$; divergent by (N.T.T.)

(30) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}$; absolutely convergent, compare with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(31) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$; divergent by (N.T.T.)

(32) $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^{n-1}$; absolutely convergent by RatioT or by G.S.

(33) $\sum_{n=1}^{\infty} \left(\frac{n}{n^2+1}\right)^n$; absolutely convergent by RootT

(34) $\sum_{n=1}^{\infty} \left(2^n + \frac{1}{2^n}\right)$; the first G.S. is divergent

(35) $\sum_{n=1}^{\infty} \frac{(-1)^n\ n}{(n+1)(n+2)}$; conditionally convergent, compare with divergent series $\sum_{n=1}^{\infty} \frac{1}{n}$

(36) $\sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi}{3}\right)}{n^3}$; absolutely convergent, compare with $\sum_{n=1}^{\infty} \frac{1}{n^3}$

(37) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n} + \sqrt{n-1}}$; conditionally convergent, compare with divergent series $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$

(38) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n(n^2+1)}}$; absolutely convergent, compare with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

(39) $\sum_{n=1}^{\infty} \frac{(-1)^n \; n!}{2^{n+1}}$; divergent by RatioT

(40) $\sum_{n=1}^{\infty} \frac{(-1)^n \; (2n+1)}{(n+2)\;\sqrt{n}}$; conditionally convergent, compare with divergent series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(41) $\sum_{n=2}^{\infty} \frac{(-1)^n \; \sqrt{n+1}}{n-1}$; conditionally convergent, compare with divergent series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(42) $\sum_{n=2}^{\infty} \frac{(-1)^{n+1} \; (n^2+1)}{n^3-1}$

conditionally convergent, compare with divergent series $\sum_{n=1}^{\infty} \frac{1}{n}$

(43) $\sum_{n=1}^{\infty} \frac{(-1)^n \; e^n}{n}$; divergent by RatioT

(44) $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{\ln(n)}{n}$; conditionally convergent, compare with divergent series $\sum_{n=1}^{\infty} \frac{1}{n}$

or use I.T.

(45) $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \; \ln(n)}$; conditionally convergent, $\sum_{n=1}^{\infty} \frac{1}{n \; \ln(n)}$ diverges by I.T.

(46) $\sum_{n=2}^{\infty} (-1)^n \left(\frac{n-1}{n} \right)$; divergent by (N.T.T.)

(47) $\sum_{n=1}^{\infty} \left(\sqrt[n]{n} \; - \; 1 \right)^n$; absolutely convergent by RootT

(48) $\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^2+1}$; absolutely convergent, compare with $\sum_{n=1}^{\infty} \frac{1}{n^2}$ or use I.T.