Math - Calculus II SERIES (2)

11.7 from Stewart (4th edition)

Test the series for convergence or divergence.

(19) $\sum_{n=0}^{\infty} \frac{n!}{2.5.8.....(3n+2)}$ convergent (absolutely) by RatioT: $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{3} < 1$

(20) $\sum_{n=1}^{\infty} \frac{(-1)^n\, n}{(n+1)(n+2)}$ use AST , series converges

(21) $\sum_{i=1}^{\infty} \frac{1}{\sqrt{i(i+1)}}$ C.T. ➜ $\sum \frac{1}{n}$; series div

(22) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$ C.T. ➜ $\sum \frac{1}{n^2}$; series conv

(23) $\sum_{n=1}^{\infty} (-1)^n\, 2^{\frac{1}{n}}$ nTT: $\lim_{n\to\infty} a_n = \pm 1 \neq 0$; series div

(24) $\sum_{n=1}^{\infty} \frac{\cos\,(n/2)}{n^2+4n}$ C.T. ➜ $\sum \frac{1}{n^2}$; series abs conv

(25) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln\, n}{\sqrt{n}}$ use AST , series converges

(26) $\sum_{n=1}^{\infty} \frac{\tan\,(1/n)}{n}$ L.C.T. ➜ $\sum \frac{1}{n^2}$; series conv

(27) $\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$ RootT : rewrite: $\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n} = \sum_{n=1}^{\infty} \frac{4^n}{n^n}$ (conv)

(28) $\sum_{n=1}^{\infty} \frac{n^2+1}{5^n}$; ratio test, series converges

(29) $\sum_{k=1}^{\infty} \frac{k\,\ln\,k}{(k+1)^3}$ C.T. ➜ $\sum \frac{1}{k^{3/2}}$; $\frac{k\,\ln\,k}{(k+1)^3} \leq \frac{k\sqrt{k}}{(k+1)^3}$ for $k \geq 1$; series conv

(30) $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ C.T. ➜ $\sum \frac{1}{n^2}$; series conv or L.C.T. or I.T.

(31) $\sum_{n=1}^{\infty} \frac{2^n}{(2n+1)!}$ ratio test, series converges

(32) $\sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j+5}$ AST ; series converges

(33) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n\sqrt{n}}$ C.T. ➜ $\sum \frac{1}{n^{3/2}}$; series conv

(34) $\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$ RootT : rewrite: $\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2^n}} = \sum_{n=1}^{\infty} \frac{(2n)^n}{(n^2)^n}$ (conv)

(35) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$ RootT : $\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right)^n = e^{-1} < 1$ (conv)

(36) $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$ very difficult! C.T. ➜ $\sum \frac{1}{n^2}$; conv

$\ln n \geq e^2$ for $n \geq 1700$ ($\ln(1700) \approx 7.44$; $e^2 \approx 7.39$)

$\frac{1}{(\ln n)^{\ln n}} \leq \frac{1}{(e^2)^{\ln n}} = \frac{1}{e^{2 \ln n}} = \frac{1}{n^2}$ for $n \geq 1700$

$\sum \frac{1}{n^2}$; conv ; $\sum \frac{1}{(\ln n)^{\ln n}}$ is also conv

(37) $\sum_{n=1}^{\infty} \left(\sqrt[n]{2} - 1\right)^n$ Root Test: series converges

(38) $\sum_{n=1}^{\infty} \left(\sqrt[n]{2} - 1\right)$ nTT fails! ; C.T. ➜ $\sum \frac{1}{n}$; series div